Statistical Power Analysis for the Behavioral Sciences

Second Edition

Jacob Cohen

Department of Psychology
New York University
New York, New York

LEA LAWRENCE ERLBAUM ASSOCIATES, PUBLISHERS
1988 Hillsdale, New Jersey Hove and London

CHAPTER 8

The Analysis of Variance

8.1 Introduction and Use

This chapter deals with an entire class of problems in tests of the equality of a set of **k** population means, where **k** equals two or more. The methods of this chapter can also be used for tests of the equality of sets of mean *differences*, as in tests of interactions. The test statistic is the **F** ratio, and the model is that of the test on means of "fixed effect" variates in the analysis of variance and covariance (Edwards, 1972; Winer, 1971; Hays, 1981). In its simplest form, it is a "one-way" ("randomized groups") design with equal **n** in each sample. The power and sample size tables in this chapter are designed for greatest simplicity in these applications (Case 0). More complicated designs involving fixed effects can also be power-analyzed with the help of these tables, as will be described below. In all cases, however, the null hypothesis states that the means or mean difference of specified ("fixed") populations are equal, or, equivalently, that "effects" defined as linear functions of means are all zero. Section 8.3.5 shows how power analysis on various tests of means, which will have been described in the context of the analysis of variance, can be performed in analogous analysis of covariance designs.

The tests here can be viewed as extensions of the tests of Chapter 2, where only two fixed population means are involved. Or, conversely, the **t** test on two means is, in fact, merely a special case of the **F** test on **k** means where $\mathbf{k} = 2$, as is detailed in most statistics textbooks. As such, the same formal model assumptions are made: that the values in the **k** populations are normally distributed and have the same variance, σ^2. It is, however, well established that moderate violations of these assumptions have generally negligible effects on the validity of null hypothesis tests and power analyses. For evidence on the issue of the "robustness" of **F** tests with regard to both Type I and Type II error in the face of assumption violation, see Scheffé (1959, Chapter 10), and for a less technical summary, Cohen (1965, pp. 114–116).[1] Note that no assumption is made about the distribution of the **k** population means for fixed effects.

The **F** test on means for fixed effects can occur under a variety of circumstances for which the tables in this chapter may be used:

Case 0. One-way analysis of variance with **n**'s equal. This is the simplest design, where without other considerations, one compares **k** means based on samples of equal size.

Case 1. One-way analysis of variance with unequal **n**'s.

Case 2. Tests of main effects in factorial and other complex designs.

Case 3. Tests of interactions in factorial designs.

Analysis of Covariance. Each of the above cases has its analog in the analysis of covariance.

8.2 The Effect Size Index: f

Our need for a pure number to index the degree of departure from no effect (i.e., **k** equal population means) is here satisfied in a way related to the solution in Chapter 2, where there were only two means. Recall that the difference in means was "standardized" by dividing it by the (common) within-population standard deviation, i.e.,

(2.2.1) $$\mathbf{d} = \frac{\mathbf{m}_1 - \mathbf{m}_2}{\sigma}.$$

Since both numerator and denominator are expressed in the (frequently arbitrary) original unit of measurement, their ratio, **d**, is a pure or dimensionless number.

With $\mathbf{k} \geq 2$ means such as we deal with here, we represent the spread of the means not by their range as above (except secondarily, see below), but by a quantity formally like a standard deviation, again dividing by the common standard deviation of the populations involved. It is thus

[1] Budescu and Applebaum (1981) have shown that when the F test is applied to samples from binomial and Poisson population distributions, the use of variance stabilizing transformations results in little change in significance level or, in most cases, power. Budescu (1982) reported that for normally distributed populations with heterogeneous variances, substituting for σ in the denominator of Equation (8.2.1) the square root of the n_i-weighted population variance results in good power approximations.

Also, Koele (1982) shows how to calculate power for random and mixed models, and demonstrates that they have much lower power than that for fixed effects. Barcikowski (1973) provides tables for optimum sample size/number of levels for the random effects model.

(8.2.1) $$\mathbf{f} = \frac{\sigma_\mathbf{m}}{\sigma},$$

where, for equal **n** (Cases 0 and 2),

(8.2.2) $$\sigma_\mathbf{m} = \sqrt{\frac{\sum_{i=1}^{\mathbf{k}} (\mathbf{m}_i - \mathbf{m})^2}{\mathbf{k}}},$$

the standard deviation of the population means expressed in original scale units. The values in the parentheses are the departures of the population means ($\mathbf{m}_i$) from the mean of the combined populations or the mean of the means for equal sample sizes (**m**), and are sometimes called the (fixed) "effects"; the σ's of formulas (8.2.1) and (2.2.1) are the same, the standard deviation within the populations, also expressed in original scale units. **f** is thus also a pure number, the standard deviation of the standardized means. That is to say that if all the values in the combined populations were to be converted into **z** "standard" scores (Hays, 1973, p. 250f), using the within-population standard deviation, **f** is the standard deviation of these **k** mean **z** scores.

f can take on values between zero, when the population means are all equal (or the effects are all zero), and an indefinitely large number as $\sigma_\mathbf{m}$ increases relative to σ.

The structure of **F** ratio tests on means, hence the index **f**, is "naturally" nondirectional (as was the index **w** of the preceding chapter). Only when there are two population means are there only two directions in which discrepancies between null and alternative hypotheses can occur. With $\mathbf{k} > 2$ means, departures can occur in many "directions." The result of all these departures from the null are included in the upper tail rejection region, and, as normally used, **F** tests do not discriminate among these and are therefore nondirectional.

f is related to an index ϕ used in standard treatments of power,[2] nomographs for which are widely reprinted in statistical testbooks (e.g., Winer, 1971; Scheffé, 1959) and books of tables (Owen, 1962). ϕ standardizes by the standard *error* of the sample mean and is thus (in part) a function of the size of each sample, **n**, while **f** is solely a descriptor of the population. Their relationship is given by

(8.2.3) $$\mathbf{f} = \frac{\phi}{\sqrt{\mathbf{n}}},$$

or

(8.2.4) $$\phi = \mathbf{f}\sqrt{\mathbf{n}}$$

[2] This use of the symbol ϕ is not to be confused with its other uses in the text, as the fourfold-point product-moment correlation in Chapter 7 or as the arcsine transformation of a proportion in Chapter 6.

The above description has, for the sake of simplicity, proceeded on the assumption that the sizes of the **k** samples are all the same. No change in the basic conception of **f** takes place when we use it to index the effect size for tests on means of samples of unequal size (Case 1) or as an ES measure for tests on interactions (Case 3). In these applications, the definition of **f** as the "standard deviation of standardized means" requires some further elaboration, which is left to the sections concerned with these cases.

The remainder of this section provides systems for the translation of **f** into (a) a range measure, **d**, and (b) correlation ratio and variance proportion measures, and offers operational definitions of "small," "medium," and "large" ES. Here, too, *the exposition proceeds on the assumption of equal* **n** *per sample* and is appropriate to the **F** test on means (Cases 0 and 2). In later discussion of Cases 1 and 3, qualifications will be offered, as necessary.

8.2.1 **f** AND THE STANDARDIZED RANGE OF POPULATION MEANS, **d**. Although our primary ES index is **f**, the standard deviation of the standardized **k** population means, it may facilitate the use and understanding of this index to translate it to and from **d**, the range of standardized means, i.e., the distance between the smallest and largest of the **k** means:

(8.2.5) $$\mathbf{d} = \frac{\mathbf{m}_{max} - \mathbf{m}_{min}}{\sigma},$$

where $\mathbf{m}_{max}$ = the largest of the **k** means,
$\mathbf{m}_{min}$ = the smallest of the **k** means, and
σ = the (common) standard deviation within the populations (as before).

Notice that in the case of **k** = 2 means (**n** equal), the **d** of (8.2.5.) becomes the **d** used as the ES index for the **t** test of Chapter 2. The relationship between **f** and **d** for 2 means is simply

(8.2.6) $$\mathbf{f} = \tfrac{1}{2}\mathbf{d},$$

i.e., the standard deviation of two values is simply half their difference, and therefore

(8.2.7) $$\mathbf{d} = 2\mathbf{f}.$$

As the number of means increases beyond two, the relationship between their standard deviation (**f**) and their range (**d**) depends upon exactly how the means are dispersed over their range. With **k** means, two (the largest and smallest) define **d**, but then the remaining **k** − 2 may fall variously over the **d** interval; thus, **f** is not uniquely determined without further specification of the pattern of separation of the means. We will identify three patterns

and describe the relationship each one has to **f**, which is also, in general, a function of the number of means. The patterns are:

1. Minimum variability: one mean at each end of **d**, the remaining **k** − 2 means all at the midpoint.
2. Intermediate variability: the **k** means equally spaced over **d**.
3. Maximum variability: the means all at the end points of **d**.

For each of these patterns, there is a fixed relationship between **f** and **d** for any given number of means, **k**.

Pattern 1. For any given range of means, **d**, the minimum standard deviation, $\mathbf{f}_1$, results when the remaining **k** − 2 means are concentrated at the mean of the means (0 when expressed in standard units), i.e., half-way between the largest and smallest. For Pattern 1,

(8.2.8) $$\mathbf{f}_1 = \mathbf{d}\sqrt{\frac{1}{2\mathbf{k}}}$$

gives the value of **f** for **k** means when the range **d** is specified. For example, 7 (= **k**) means dispersed in Pattern 1 would have the (standardized) values $-\tfrac{1}{2}\mathbf{d}$, 0, 0, 0, 0, 0, $+\tfrac{1}{2}\mathbf{d}$. Their standard deviation would be

$$\mathbf{f}_1 = \mathbf{d}\sqrt{\frac{1}{2(7)}} = \sqrt{.071429} = .267\mathbf{d},$$

slightly more than one-quarter of the range. Thus, a set of 7 population means spanning half a within-population standard deviation would have **f** = .267(.5) = .13.

The above gives **f** as a function of **d**. The reciprocal relationship is required to determine what value of the range is implied by any given (e.g., tabled) value of **f** when Pattern 1 holds, and is

(8.2.9) $$\mathbf{d}_1 = \mathbf{f}\sqrt{2\mathbf{k}}.$$

For example, for the 7 (= **k**) means dispersed in Pattern 1 above, their range would be

$$\mathbf{d}_1 = \mathbf{f}\sqrt{2(7)} = \mathbf{f}\sqrt{14} = 3.74\mathbf{f}.$$

A value of **f** = .50 for these means would thus imply a standardized range of 3.74(.50) = 1.87.

For the convenience of the user of this handbook, Table 8.2.1 gives the constants (**c** and **b**) relating **f** to **d** for this pattern and the others discussed below for **k** = 2(1) 16, 25, covering the power and sample size tables provided. Their use is illustrated later in the chapter.

Table 8.2.1
Constants for Transforming d to f_j and f to d_j for Patterns j = 1, 2, 3

	$f_j = c_j d$			$d_j = b_j f$		
k	c_1	c_2	c_3	b_1	b_2	b_3
2	.500	.500	.500	2.00	2.00	2.00
3	.408	.408	.471	2.45	2.45	2.12
4	.354	.373	.500	2.83	2.68	2.00
5	.316	.354	.490	3.16	2.83	2.04
6	.289	.342	.500	3.46	2.93	2.00
7	.267	.333	.495	3.74	3.00	2.02
8	.250	.327	.500	4.00	3.06	2.00
9	.236	.323	.497	4.24	3.10	2.01
10	.224	.319	.500	4.47	3.13	2.00
11	.213	.316	.498	4.69	3.16	2.01
12	.204	.314	.500	4.90	3.19	2.00
13	.196	.312	.499	5.10	3.21	2.01
14	.189	.310	.500	5.29	3.22	2.00
15	.183	.309	.499	5.48	3.24	2.00
16	.177	.307	.500	5.66	.325	2.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮
25	.141	.300	.500	7.07	3.01	2.00

Pattern 2. A pattern of medium variability results when the **k** means are equally spaced over the range, and therefore at intervals of $\mathbf{d}/(\mathbf{k}-1)$. For Pattern 2, the **f** which results from any given range **d** is

(8.2.10)
$$\mathbf{f}_2 = \frac{\mathbf{d}}{2}\sqrt{\frac{\mathbf{k}+1}{3(\mathbf{k}-1)}}.$$

For example, for $\mathbf{k} = 7$,

$$\mathbf{f}_2 = \frac{\mathbf{d}}{2}\sqrt{\frac{7+1}{3(7-1)}} = \frac{\mathbf{d}}{2}\sqrt{\frac{8}{18}} = .333\mathbf{d},$$

i.e., 7 equally spaced means would have the values $-\frac{1}{2}\mathbf{d}$, $-\frac{1}{3}\mathbf{d}$, $-\frac{1}{6}\mathbf{d}$, 0, $+\frac{1}{6}\mathbf{d}$, $+\frac{1}{3}\mathbf{d}$, and $+\frac{1}{2}\mathbf{d}$, and a standard deviation equal to one-third of their range.

Note that this value for the same **k** is larger than $\mathbf{f}_1 = .267\mathbf{d}$ for Pattern 1. For a range of half a within-population standard deviation, $\mathbf{f}_2 = .333(.5) = .17$ (while comparably, $\mathbf{f}_1 = .13$).

The reciprocal relationship for determining the range implied by a tabled (or any other) value of **f** for Pattern 2 is

(8.2.11)
$$\mathbf{d}_2 = 2\mathbf{f}\sqrt{\frac{3(\mathbf{k}-1)}{\mathbf{k}+1}}.$$

For 7 means in Pattern 2, their range would be

$$\mathbf{d}_2 = 2\mathbf{f}\sqrt{\frac{3(7-1)}{7+1}} = 2\mathbf{f}\sqrt{\frac{18}{8}} = 3\mathbf{f}.$$

Thus, a value of $\mathbf{f} = .50$ for these equally spaced means would imply a standardized range of $3(.50) = 1.50$.

Table 8.2.1 gives the relevant constants ($\mathbf{b}_2$ and $\mathbf{c}_2$) for varying **k**, making the solution of formulas (8.2.10) and (8.2.11) generally unnecessary.

Pattern 3. It is demonstrable and intuitively evident that for any given range the dispersion which yields the maximum standard deviation has the **k** means falling at both extremes of the range. When **k** is even, $\frac{1}{2}\mathbf{k}$ fall at $-\frac{1}{2}\mathbf{d}$ and the other $\frac{1}{2}\mathbf{k}$ fall at $+\frac{1}{2}\mathbf{d}$; when **k** is odd, $(\mathbf{k}+1)/2$ of the means fall at either end and the $(\mathbf{k}-1)/2$ remaining means at the other. With this pattern, for all *even* numbers of means,

(8.2.12)
$$\mathbf{f}_3 = \tfrac{1}{2}\mathbf{d}.$$

When **k** is odd, and there is thus one more mean at one extreme than at the other,

(8.2.13)
$$\mathbf{f}_3 = \mathbf{d}\frac{\sqrt{\mathbf{k}^2-1}}{2\mathbf{k}}.$$

For example, for $\mathbf{k} = 7$ means in Pattern 3 (4 means at either $-\frac{1}{2}\mathbf{d}$ or $+\frac{1}{2}\mathbf{d}$, 3 means at the other), their standard deviation is

$$\mathbf{f}_3 = \mathbf{d}\frac{\sqrt{7^2-1}}{2(7)} = \mathbf{d}\frac{\sqrt{48}}{14} = .495\mathbf{d}.$$

Note that $\mathbf{f}_3$ is larger (for $\mathbf{k} = 7$) than $\mathbf{f}_2 = .333\mathbf{d}$ and $\mathbf{f}_1 = .267\mathbf{d}$. If, as before, we posit a range of half a within-population standard deviation, $\mathbf{f}_3 = .495(.5) = .25$.

The reciprocal relationship used to determine the range implied by a given value of **f** when **k** is even is simply

(8.2.14)
$$\mathbf{d}_3 = 2\mathbf{f},$$

and when **k** is odd,

(8.2.15) $$\mathbf{d}_3 = \mathbf{f}\frac{2\mathbf{k}}{\sqrt{\mathbf{k}^2 - 1}}.$$

For the running example of $\mathbf{k} = 7$ means, in Pattern 3 their range would be

$$\mathbf{d}_3 = \mathbf{f}\frac{2(7)}{\sqrt{7^2 - 1}} = \mathbf{f}\frac{14}{\sqrt{48}} = 2.02\mathbf{f},$$

so that if we posit, as before, a value of $\mathbf{f} = .50$, for these 7 extremely placed means, $\mathbf{d}_3 = 2.02(.5) = 1.01$, i.e., slightly more than a within-population standard deviation.

As can be seen from Table 8.2.1, there is not as much variability as a function of **k** in the relationship between **f** and **d** for Pattern 3 as for the others. $\mathbf{f}_3$ is either (for **k** even) exactly or (for **k** odd) approximately $\frac{1}{2}\mathbf{d}$, the minimum value being $\mathbf{f}_3 = .471\mathbf{d}$ at $\mathbf{k} = 3$.

This section has described and tabled the relationship between the primary ES index for the **F** test, **f**, the standard deviation of standardized means, and **d**, the standardized range of means, for three patterns of distribution of the **k** means. This makes it possible to use **d** as an alternate index of effect size, or equivalently, to determine the **d** implied by tabled or other values of **f**, and **f** implied by specified values of **d**. (The use of **d** will be illustrated in the problems of Sections 8.3 and 8.4) The reader is reminded that these relationships hold only for equal sample sizes (Cases 0 and 2).

8.2.2 **f**, THE CORRELATION RATIO, AND PROPORTION OF VARIANCE. Expressing **f** in terms of **d** provides one useful perspective on the appraisal of effect size with multiple means. Another frame of reference in which to understand **f** is described in this section, namely, in terms of correlation between population membership and the dependent variable, and in the related terms of the proportion of the total variance (PV) of the **k** populations combined which is accounted for by population membership.

Just as the **d** of this chapter is a generalization to **k** populations of the **d** used as an ES index for **t** tests on two means of Chapter 2, so is η (eta), the correlation ratio, a similar generalization of the Pearson **r**, and η^2 a generalization of $\mathbf{r}^2$, the proportion of variance (PV) accounted for by population membership.

To understand η^2, consider the set of **k** populations, all of the same variance, σ^2, but each with its own mean, $\mathbf{m}_i$. The variance of the means $\sigma_\mathbf{m}^2$ is some quantity which differs from zero when the **k** means are not all equal. If we square both sides of formula (8.2.1), we note that

(8.2.16) $$\mathbf{f}^2 = \frac{\sigma_\mathbf{m}^2}{\sigma^2},$$

is the ratio of the variance of the means to the variance of the values within the populations.

Now consider that the populations are combined into a single "superpopulation" whose mean is **m** (the mean of the population $\mathbf{m}_i$'s when the populations are considered equally numerous; otherwise, their mean when each $\mathbf{m}_i$ is weighted by its population size). The variance of the "superpopulation," or total variance (σ_t^2), is larger than the within-population variance because it is augmented by the variance of the constituent population means. It is simply the sum of these two variances:

(8.2.17) $$\sigma_t^2 = \sigma^2 + \sigma_\mathbf{m}^2.$$

We now define η^2 as the proportion of the total superpopulation variance made up by the variance of the population means:

(8.2.18) $$\eta^2 = \frac{\sigma_\mathbf{m}^2}{\sigma_t^2} = \frac{\sigma_\mathbf{m}^2}{\sigma^2 + \sigma_\mathbf{m}^2}.$$

The combination of this formula with formula (8.2.16) and some simple algebraic manipulation yields

(8.2.19) $$\eta^2 = \frac{\mathbf{f}^2}{1 + \mathbf{f}^2},$$

and

(8.2.20) $$\eta = \sqrt{\frac{\mathbf{f}^2}{1 + \mathbf{f}^2}}.$$

Thus, a simple function of $\mathbf{f}^2$ yields η^2, a measure of dispersion of the $\mathbf{m}_i$ and hence of the implication of difference in population membership to the overall variability. When the population means are all equal, $\sigma_\mathbf{m}^2$ and hence $\mathbf{f}^2$ is zero, and $\eta^2 = 0$, indicating that none of the total variance is due to difference in population membership. As formula (8.2.18) makes clear, when all the cases in each population have the same value, $\sigma^2 = 0$, and all of the total variance is produced by the variance of the means, so that $\eta^2 = 1.00$. Table 8.2.2 provides η^2 and η values as a function of **f**.

Note that η^2, like all measures of ES, describes a population state of affairs. It can also be computed on samples and its population value estimated therefrom. (See examples 8.17 and 8.19.) Depending on the basis of the estimation, the estimate is variously called η^2, ϵ^2 (Peters and Van Voorhis, 1940, pp. 312–325, 353–357; Cureton, 1966, pp. 605–607), or estimated ω^2 (Hays, 1981, pp. 349–366). In general, η^2 is presented in applied statistics textbooks only in connection with its use in the appraisal of the curvilinear regression of **Y** on **X**, where the populations are defined by equal segments along the **X** variable, and $\sigma_m{}^2$ is the variance of the **X**-segments' **Y** means. Although this is a useful application of η^2, it is a rather limited special case. For the broader view, see Hays (1973) (under ω^2), Cohen (1965, pp. 104–105), Cohen & Cohen (1983, pp. 196–198) and Friedman (1968, 1982).

η^2 is literally a generalization of the (point-biserial) $\mathbf{r}^2$ of Chapter 2 which gives the PV for the case where there are $\mathbf{k} = 2$ populations. It is possible to express the relationship between the dependent variable **Y** and population membership **X** as a simple (i.e., zero-order) product moment $\mathbf{r}^2$, when **X** is restricted to two possibilities, i.e., membership in A ($\mathbf{X} = 0$) or membership in B ($\mathbf{X} = 1$) (see Chapter 2). When we generalize **X** to represent a nominal scale of **k** possible alternative population memberships, $\mathbf{r}^2$ no longer suffices, and the more general η^2 is used. It is interesting to note that if **k**-population membership is rendered as a set of independent variables (say, as dichotomous "dummy" variables), the simple $\mathbf{r}^2$ generalizes to *multiple* $\mathbf{R}^2$, which is demonstrably equal to η^2 (see Section 9.2.1).

We have interpreted η^2 as the PV associated with alternative membership in populations. A mathematically equivalent description of η^2 proceeds by the following contrast: Assume that we "predict" all the members of our populations as having the same **Y** value, the **m** of our superpopulation. The gross error of this "prediction" can be appraised by finding for each subject the discrepancy between his value and **m**, squaring this value, and adding such squared values over all subjects. Call this $\mathbf{E_t}$. Another "prediction" can be made by assigning to each subject the mean of *his* population, $\mathbf{m_i}$. Again, we determine the discrepancy between his actual value and this "prediction" ($\mathbf{m_i}$), square and total over all subjects from all populations. Call this $\mathbf{E_p}$. To the extent to which the **k** population means are spread, $\mathbf{E_p}$ will be smaller than $\mathbf{E_t}$.

(8.2.21) $$\eta^2 = \frac{\mathbf{E_t} - \mathbf{E_p}}{\mathbf{E_t}} = 1 - \frac{\mathbf{E_p}}{\mathbf{E_t}},$$

i.e., the proportionate amount *by which* errors are reduced by using own population mean ($\mathbf{m_i}$) rather than superpopulation mean (**m**) as a basis for "prediction." Or, we can view these as alternative means of *characterizing*

the members of our populations, and η^2 indexes the degree of increased incisiveness that results from using the $\mathbf{m_i}$ rather than **m**.

The discussion has thus far proceeded with η^2, the PV measure. For purposes of morale, and to offer a scale which is comparable to that of the familiar product moment **r**, we can index ES by means of η, the correlation ratio, in addition to or instead of the lower value yielded by η^2. As can be seen from taking the square root in formula (8.2.18),, η is the ratio of the *standard deviation* of population means to the *standard deviation* of the values in the superpopulation, i.e., the combined populations. Since standard devia-

Table 8.2.2

η^2 and η as a Function of f; f as a Function of η^2 and η

f	η^2	η	η^2	f	η	f
.00	.0000	.000	.00	.000	.00	.000
.05	.0025	.050	.01	.101	.05	.050
.10	.0099	.100	.02	.143	.10	.101
.15	.0220	.148	.03	.176	.15	.152
.20	.0385	.196	.04	.204	.20	.204
.25	.0588	.243	.05	.229	.25	.258
.30	.0826	.287	.06	.253	.30	.314
.35	.1091	.330	.07	.274	.35	.374
.40	.1379	.371	.08	.295	.40	.436
.45	.1684	.410	.09	.314	.45	.504
.50	.2000	.447	.10	.333	.50	.577
.55	.2322	.482	.15	.420	.55	.659
.60	.2647	.514	.20	.500	.60	.750
.65	.2970	.545	.25	.577	.65	.855
.70	.3289	.573	.30	.655	.70	.980
.75	.3600	.600	.40	.816	.75	1.134
.80	.3902	.625	.50	1.000	.80	1.333
.85	.4194	.648	.60	1.225	.85	1.614
.90	.4475	.669	.70	1.528	.90	2.065
.95	.4744	.689	.80	2.000	.95	3.042
1.00	.5000	.707	.90	3.000	1.00	–

tions are as respectable as variances, no special apology is required in working with η rather than η^2.

In formulas (8.2.19) and (8.2.20), we have η^2 and η as functions of **f**. This is useful for assessing the implication of a given value of **f** (in terms of which our tables are organized) to PV or correlation. The reciprocal relation, **f** as a function of η, is also useful when the investigator, thinking in PV or correlational terms, needs to determine the **f** they imply, e.g., in order to use the tables:

(8.2.22) $$\mathbf{f} = \sqrt{\frac{\eta^2}{1-\eta^2}}$$

For the convenience of the user of this handbook, this formula is solved for various values of η and η^2 and the results presented in Table 8.2.2.

Table 8.2.2 deserves a moment's attention. As discussed in the next section and in Section 11.1 (and, indeed, as noted in previous chapters, particularly Chapter 3), effect sizes in behavioral science are generally small, and, in terms of **f**, will generally be found in the .00–.40 range. With **f** small, $\mathbf{f}^2$ is smaller, and $1 + \mathbf{f}^2$, the denominator of η^2 [formula (8.2.19)] is only slightly greater than one. The result is that for small values of **f** such as are typically encountered, η is approximately equal to **f**, being only slightly smaller, and therefore η^2 is similarly only slightly smaller than $\mathbf{f}^2$. Thus, in the range of our primary interest, **f** provides in itself an approximate correlation measure, and $\mathbf{f}^2$ an approximate PV measure. For very large effect sizes, say $\mathbf{f} > .40$, **f** and η diverge too much for this rough and ready approximation, and $\mathbf{f}^2$ and η^2 even more so.

8.2.3 "SMALL," "MEDIUM," AND "LARGE" **f** VALUES. It has already been suggested that values of **f** as large as .50 are not common in behavioral science, thus providing a prelude to the work of this section. Again, as in previous chapters, we take on the task of helping the user of this handbook to achieve a workable frame of reference for the ES index or measure of the alternate-hypothetical state of affairs, in this case **f**.

The optimal procedure for setting **f** in a given investigation is that the investigator, drawing on previous findings and theory in that area and his own scientific judgment, specify the **k** means and σ he expects and compute the resulting **f** from these values by means of formulas (8.2.1) and (8.2.2). If this demand for specification is too strong, he may specify the range of means, **d**, from formula (8.2.5), choose one of the patterns of mean dispersion of Section 8.2.1, and use Table 8.2.1 to determine the implied value of **f**. On the same footing as this procedure, which may be used instead of or in conjunction with it, is positing the expected results in terms of the proportion of total variance associated with membership in the **k** populations, i.e., η^2. Formula (8.2.22) and Table 8.2.2 then provide the translation from η^2 to **f**. (In the case of **f** for interactions, see Section 8.3.4.)

All the above procedures are characterized by their use of magnitudes selected by the investigator to represent the situation of the *specific* research he is planning. When experience with a given research area or variable is insufficient to formulate alternative hypotheses as "strong" as these procedures demand, and to serve as a set of conventions or operational definitions, we define specific values of **f** for "small," "medium," and "large" effects. The reader is referred to Sections 1.4 and 2.2.3 for review of the considerations leading to the setting of ES conventions, and the advantages and disadvantages inherent in them. Briefly, we note here that these qualitative adjectives are relative, and, being general, may not be reasonably descriptive in any specific area. Thus, what a sociologist may consider a small effect size may well be appraised as medium by a clinical psychologist.

It must be reiterated here that however problematic the setting of an ES, it is a task which simply cannot be shirked. The investigator who insists that he has absolutely no way of knowing how large an ES to posit fails to appreciate that this necessarily means that he has no rational basis for deciding whether he needs to make ten observations or ten thousand.

Before presenting the operational definitions for **f**, a word about their consistency. They are fully consistent with the definitions of Chapter 2 for $\mathbf{k} = 2$ populations in terms of **d**, which, as noted, is simply 2**f**. They are also generally consistent with the other ES indices which can be translated into PV measures (see Sections 3.2.2 and 6.2.1).

We continue, for the present, to conceive of the populations as being sampled with equal **n**'s.

SMALL EFFECT SIZE: $\mathbf{f} = .10$. We define a small effect as a standard deviation of **k** population means one-tenth as large as the standard deviation of the observations within the populations. For $\mathbf{k} = 2$ populations, this definition is exactly equivalent to the comparable definition of a small difference, $\mathbf{d} = 2(.10) = .20$ of Chapter 2 [formula (8.2.7) and, more generally, Table 8.2.1]. As **k** increases, a given **f** implies a greater range for Patterns 1 and 2. Thus, with $\mathbf{k} = 6$ means, one at each end of the range and the remaining 4 at the middle (Pattern 1), an **f** of .10 implies a range $\mathbf{d}_1$ of $3.46(.10) = .35$, while equal spacing (Pattern 2) implies a range $\mathbf{d}_2$ of $2.93(.10) = .29$. (The constants 3.46 and 2.93 are respectively the $\mathbf{b}_1$ and $\mathbf{b}_2$ values at $\mathbf{k} = 6$ in Table 8.2.1.) When $\mathbf{f} = .10$ occurs with the extreme Pattern 3, the $\mathbf{d}_3$ is at (for **k** even) or slightly above (for **k** odd) $2\mathbf{f} = .20$ (Table 8.2.1). Thus, depending on **k** and the pattern of the means over the range, a small effect implies **d** at least .20, and, with large **k** disposed in Pattern 1, a small effect can be expressed in a $\mathbf{d}_1$ of the order of .50 or larger (for example, see Table 8.2.1 in column $\mathbf{b}_1$ for $\mathbf{k} \geq 12$).

When expressed in correlation and PV terms, the $\mathbf{f} = .10$ definition of a small effect is fully consistent with the definitions of Chapters 2, 3, and 6 (various forms of product moment $\mathbf{r}$). An $\mathbf{f} = .10$ is equivalent to $\eta = .100$ and $\eta^2 = .0099$, about 1% of the total superpopulation variance accounted for by group membership. As already noted (particularly in Section 2.2.3), scientifically important (or at least meaningful) effects may be of this modest order of magnitude. The investigator who is inclined to disregard ES criteria for effects this small on the grounds that he would never be seeking to establish such small effects needs to be reminded that he is likely to be thinking in terms of theoretical constructs, which are implicitly measured without error. Any source of irrelevant variance in his measures (psychometric unreliability, dirty test tubes, lack of experimental control, or whatever) will serve to reduce his effect sizes *as measured*, so that what would be a medium or even large effect if one could use "true" measures may be attenuated to a small effect in practice (See Section 11.3 and Cohen, 1962, p. 151).

MEDIUM EFFECT SIZE: $\mathbf{f} = .25$. A standard deviation of $\mathbf{k}$ population means one-quarter as large as the standard deviation of the observations within the populations, is the operational definition of a medium effect size. With $\mathbf{k} = 2$ populations, this accords with the $\mathbf{d} = 2(.25) = .50$ definition of a medium difference between two means of Chapter 2, and this is a minimum value for the range over $\mathbf{k}$ means. With increasing $\mathbf{k}$ for either minimum (Pattern 1) or intermediate (Pattern 2) variability, the range implied by $\mathbf{f} = .25$ increases from $\mathbf{d} = .50$. For example, with $\mathbf{k} = 7$ population means, if $\mathbf{k} - 2 = 5$ of them are at the middle of the range and the remaining two at the endpoints of the range (Pattern 1), a medium $\mathbf{d}_1 = 3.74(.25) = .94$ (Table 8.2.1 gives $\mathbf{b}_1 = 3.74$ at $\mathbf{k} = 7$). Thus, medium effect size for 7 means disposed in Pattern 1 implies a range of means of almost one standard deviation. If the seven means are spaced equally over the range (Pattern 2), a medium $\mathbf{d}_2 = 3.00(.25) = .75$ (Table 8.2.1 gives $\mathbf{b}_2 = 3.00$ for $\mathbf{k} = 7$), i.e., a span of means of three-quarters of a within-population standard deviation. As a concrete example of this, consider the IQ's of seven populations made up of certain occupational groups, e.g., house painters, chauffeurs, auto mechanics, carpenters, butchers, riveters, and linemen. Assume a within-population standard deviation for IQ of 12 ($=\sigma$) and that their IQ means are equally spaced. Now, assume a medium ES, hence $\mathbf{f} = .25$. (Expressed in IQ units, this would mean that the standard deviation of the seven IQ means would be $\mathbf{f}\sigma = .25(12) = 3$.) The range of these means would be $\mathbf{d}_2 = .75$ of the within-population σ. Expressed in units of IQ, this would be $\mathbf{d}_2\sigma = .75(12) = 9$ IQ points, say from 98 to 107. (These values are about right [Berelson & Steiner, 1964, pp. 223–224], but of course any seven equally spaced values whose range is 9 would satisfy the criterion of a medium ES as defined here.)

Viewed from the perspective of correlation and proportion of variance accounted for, we note that $\mathbf{f} = .25$ implies a correlation ratio (η) of .243 and a PV (here η^2) of .0588, i.e., not quite 6% of the total variance of the combined populations accounted for by population membership (Table 8.2.2). Again, note that this is identical with the correlational-PV criterion of a medium difference between two means (Section 2.2), necessarily so since in this limiting case $\eta = \mathbf{r}$ (point biserial). It is also consistent with the definition of a medium difference between two proportions, when expressed as an $\mathbf{r}$ (fourfold point or ϕ correlation), which equals .238 to .248 when, the proportions are in the interval .20 to .80 (Section 6.2). It is, however smaller than the criterion for a medium ES in hypotheses concerning the Pearson $\mathbf{r}$ (Section 3.2), where the medium $\mathbf{r}$ is .30 (and $\mathbf{r}^2 = .09$).

LARGE EFFECT SIZE: $\mathbf{f} = .40$. Our operational definition (or proposed convention) of a large spread of $\mathbf{k}$ means is that the standard deviation of the means be .40 of the standard deviation of the observations within the populations. This is consistent with the criterion of a large difference between two means of $\mathbf{d} = 2(.40) = .80$ (Section 2.2.2) and is the minimum range (since $\mathbf{k} = 2$) which can be called large by this definition. With the means disposed in Pattern 1, a large span for 6 means is $\mathbf{d}_1 = 3.46(.40) = 1.38$, for 7 means $\mathbf{d}_1 = 3.74(.40) = 1.50$, for 8 means $\mathbf{d}_1 = 4.00(.40) = 1.60$, etc., i.e., about $1\frac{1}{2}$ standard deviations ($\mathbf{b}_1$ constants from Table 8.2.1). For equally spaced means (Pattern 2), this implies for 6 means, a range of $\mathbf{d}_2 = 2.93(.40) = 1.17$, for 7 means a range of $\mathbf{d}_2 = 3.00(.40) = 1.20$, and for 8 means a range of $\mathbf{d}_2 = 3.06(.40) = 1.22$, etc., i.e., about $1\frac{1}{4}$ standard deviations ($\mathbf{b}_2$ constants from Table 8.2.1). We use a similar illustration to that given for medium effect size, where for $\mathbf{k} = 7$ occupation groups with equally spaced population mean IQs, we found the range $\mathbf{d}_2 = \mathbf{b}_2\mathbf{f} = 3.00(.25) = .75$, or, expressed in IQ units, $.75\sigma = .75(12) = 9.0$. Consider now a new set of 7 occupations: house painter, chauffeur, upholsterer, mechanic, lathe operator, machinist, laboratory assistant. Their mean IQ's, to have a large range, would need to cover uniformly the interval $\mathbf{d}_2 = \mathbf{b}_2\mathbf{f} = 3.00(.40) = 1.20$, or expressed in IQ units, again assuming that $\sigma = 12$, $1.20\sigma = 1.20(12) = 14.4$, say from 98 to 112 (Berelson & Steiner, 1964, pp. 223–224). Again note that any set of 7 occupation groups with IQ means spanning the same range would represent a large effect as defined here, wherever that range occurs.

In terms of correlation and proportion of variance accounted for, $\mathbf{f} = .40$ implies a correlation ratio (η) of .371 and a PV (here η^2) of .1379, somewhat more than twice the PV for a medium effect ($\eta^2 = .0588$). Note the necessary consistency with the definition in correlation–PV terms of a large difference between two means (η = point biserial $\mathbf{r}$; see Section 2.2). This definition is also fully consistent with the definition of a large difference between two proportions, when expressed as an $\mathbf{r}$ (fourfold point or ϕ

correlation), which equals .37–.39 when the proportions fall between .20 and .80 (Section 6.2). However, it is smaller than the criterion for a large ES in hypotheses concerning the Pearson **r**, where large **r** is defined as .50, $r^2 =$ PV = .25 (Section 3.2).

8.3 POWER TABLES

The power tables for this section are given on pages 289–354; the text follows on page 355.

Table 8.3.1

Power of F test at a = .01, u = 1

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	98.503	01	01	01	01	02	02	03	04	04	05	06	08
3	21.198	01	01	01	02	02	02	03	04	05	07	09	11
4	13.745	01	01	01	02	02	03	04	05	07	10	14	19
5	11.259	01	01	02	02	03	03	05	06	10	15	21	29
6	10.044	01	01	02	02	03	04	06	08	13	20	29	40
7	9.330	01	01	02	03	04	05	07	10	17	26	38	50
8	8.861	01	01	02	03	04	06	09	12	21	32	46	60
9	8.531	01	02	02	03	05	07	10	14	25	39	54	68
10	8.285	01	02	02	04	06	08	12	17	29	45	61	75
11	8.096	01	02	03	04	06	09	14	19	34	51	67	81
12	7.946	01	02	03	05	07	11	16	22	38	56	73	86
13	7.823	01	02	03	05	08	12	18	25	42	61	78	89
14	7.721	01	02	03	05	08	13	20	28	46	66	82	92
15	7.636	01	02	03	06	09	15	22	30	50	70	85	94
16	7.562	01	02	04	06	10	16	24	33	54	74	88	96
17	7.499	01	02	04	07	11	17	26	36	58	78	91	97
18	7.444	01	02	04	07	12	19	28	39	62	81	92	98
19	7.396	01	02	04	08	13	20	30	41	65	83	94	98
20	7.353	01	02	04	08	14	22	32	44	68	86	95	99
21	7.314	01	02	05	08	15	24	34	47	71	88	96	99
22	7.280	01	03	05	09	16	25	37	49	73	90	97	99
23	7.248	01	03	05	09	17	27	39	52	76	91	98	*
24	7.220	01	03	05	10	18	28	41	54	78	93	98	
25	7.194	01	03	06	10	19	30	43	57	80	94	99	
26	7.171	01	03	06	11	20	31	45	59	82	95	99	
27	7.149	01	03	06	12	21	33	47	61	84	96	99	
28	7.129	01	03	06	12	22	35	49	63	86	96	99	
29	7.110	01	03	07	13	23	36	50	65	87	97	*	
30	7.093	01	03	07	13	24	38	53	67	89	97		
31	7.077	02	03	07	14	25	39	55	69	90	98		
32	7.062	02	03	07	15	26	41	56	71	91	98		
33	7.048	02	04	08	15	27	42	58	73	92	99		
34	7.035	02	04	08	16	28	44	60	75	93	99		
35	7.023	02	04	08	17	30	45	62	76	94	99		
36	7.011	02	04	08	17	31	47	63	78	94	99		
37	7.001	02	04	09	18	32	48	65	79	95	99		
38	6.990	02	04	09	19	33	50	66	80	96	99		
39	6.981	02	04	09	19	34	51	68	82	96	*		

Table 8.3.1 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	6.971	02	04	10	20	35	53	69	83	97	*	*	*
42	6.954	02	04	10	21	37	55	72	85	97			
44	6.939	02	05	11	23	39	58	75	87	98			
46	6.925	02	05	11	24	41	60	77	89	98			
48	6.912	02	05	12	25	44	63	79	90	99			
50	6.901	02	05	13	27	46	65	81	92	99			
52	6.890	02	05	13	28	48	67	83	93	99			
54	6.880	02	06	14	30	50	70	85	94	99			
56	6.871	02	06	15	31	52	72	86	95	*			
58	6.862	02	06	16	33	54	73	88	95				
60	6.854	02	06	16	34	56	75	89	96				
64	6.840	02	07	18	37	59	79	91	97				
68	6.828	02	07	19	40	63	82	93	98				
72	6.817	02	08	21	42	66	84	95	99				
76	6.807	02	08	22	45	69	87	96	99				
80	6.798	02	09	24	48	72	89	97	99				
84	6.790	03	09	25	50	74	90	97	*				
88	6.783	03	10	27	53	77	92	98					
92	6.776	03	10	29	55	79	93	98					
96	6.770	03	11	30	57	81	94	99					
100	6.764	03	11	32	60	83	95	99					
120	6.742	03	14	40	70	90	98	*					
140	6.727	04	17	47	78	95	99						
160	6.715	04	21	54	84	97	*						
180	6.706	04	24	61	89	99							
200	6.699	05	28	67	92	99							
250	6.686	07	37	79	97	*							
300	6.677	08	45	87	99								
350	6.671	10	53	92	*								
400	6.667	11	60	95									
450	6.663	13	67	97									
500	6.661	15	73	99									
600	6.656	19	82	*									
700	6.653	24	88										
800	6.651	28	93										
900	6.649	32	95										
1000	6.648	37	97										

* Power values below this point are greater than .995.

Table 8.3.2

Power of F test at a = .01, u = 2

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	30.817	01	01	01	01	02	02	03	03	03	04	06	07
3	10.925	01	01	01	02	02	02	03	04	05	07	10	13
4	8.022	01	01	01	02	02	03	04	05	08	12	17	24
5	6.927	01	01	02	02	03	04	05	07	11	18	27	38
6	6.359	01	01	02	02	03	05	07	09	16	26	38	51
7	6.013	01	01	02	03	04	06	08	11	21	33	48	63
8	5.780	01	01	02	03	05	07	10	14	26	41	58	73
9	5.614	01	02	02	04	05	08	12	17	31	49	67	81
10	5.488	01	02	03	04	06	10	14	21	37	56	74	87
11	5.390	01	02	03	04	07	11	17	24	42	63	80	91
12	5.313	01	02	03	05	08	13	19	27	48	69	85	94
13	5.249	01	02	03	05	09	14	22	31	53	74	89	96
14	5.195	01	02	03	06	10	16	24	34	58	79	92	98
15	5.150	01	02	04	06	11	18	27	38	62	82	94	99
16	5.111	01	02	04	07	12	20	30	41	67	86	96	99
17	5.078	01	02	04	07	13	21	32	45	70	89	97	99
18	5.048	01	02	04	08	14	23	35	48	74	91	98	*
19	5.022	01	02	05	09	15	25	38	52	77	93	98	
20	4.999	01	02	05	09	17	27	40	55	80	94	99	
21	4.977	01	03	05	10	18	29	43	58	83	95	99	
22	4.959	01	03	05	10	19	31	45	61	85	96	*	
23	4.943	01	03	06	11	20	33	48	64	87	97		
24	4.928	01	03	06	12	22	35	51	66	89	98		
25	4.914	01	03	06	12	23	37	53	69	91	98		
26	4.901	01	03	07	13	24	39	56	71	92	99		
27	4.889	01	03	07	14	26	41	58	74	93	99		
28	4.878	01	03	07	15	27	43	60	75	94	99		
29	4.868	01	03	07	15	28	45	62	78	95	99		
30	4.859	02	03	08	16	30	47	65	80	96	*		
31	4.850	02	04	08	17	31	49	67	81	96			
32	4.842	02	04	08	18	33	51	69	83	97			
33	4.834	02	04	09	19	34	53	70	84	98			
34	4.827	02	04	09	19	35	54	72	86	98			
35	4.820	02	04	09	20	37	56	74	87	98			
36	4.814	02	04	10	21	38	58	76	88	99			
37	4.808	02	04	10	22	40	59	77	89	99			
38	4.802	02	04	10	23	41	61	79	90	99			
39	4.797	02	04	11	24	42	63	80	91	99			

Table 8.3.2 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	4.791	02	05	11	25	44	64	81	92	99	*	*	*
42	4.782	02	05	12	26	46	67	84	94	*			
44	4.774	02	05	13	28	49	70	86	95				
46	4.766	02	05	14	30	51	73	88	96				
48	4.760	02	05	14	32	54	75	90	97				
50	4.753	02	06	15	33	56	77	91	97				
52	4.747	02	06	16	35	59	79	92	98				
54	4.742	02	06	17	37	61	81	93	98				
56	4.737	02	06	18	39	63	83	94	99				
58	4.732	02	07	19	40	65	85	95	99				
60	4.728	02	07	20	42	67	86	96	99				
64	4.720	02	08	22	46	71	89	97	99				
68	4.713	02	08	24	49	75	91	98	*				
72	4.707	02	09	26	52	78	93	99					
76	4.702	02	09	28	55	81	95	99					
80	4.697	03	10	30	58	83	96	99					
84	4.693	03	10	32	61	85	97	*					
88	4.689	03	11	34	64	88	97						
92	4.685	03	12	36	67	89	98						
96	4.682	03	13	38	69	91	98						
100	4.678	03	13	40	72	92	99						
120	4.666	04	17	49	82	97	*						
140	4.657	04	21	58	89	99							
160	4.651	05	26	66	93	99							
180	4.646	05	30	73	96	*							
200	4.642	06	34	79	98								
250	4.634	07	45	89	99								
300	4.629	09	56	95	*								
350	4.626	11	65	97									
400	4.623	13	72	99									
450	4.621	16	79	*									
500	4.620	18	84										
600	4.617	24	91										
700	4.616	29	95										
800	4.614	35	98										
900	4.613	40	99										
1000	4.612	46	99										

* Power values below this point are greater than .995.

Table 8.3.3

Power of F test at a = .01, u = 3

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	16.694	01	01	01	01	02	02	02	03	04	05	06	07
3	7.591	01	01	01	02	02	03	03	04	06	08	12	16
4	5.953	01	01	01	02	02	03	04	06	09	15	22	31
5	5.292	01	01	02	02	03	04	06	08	14	23	34	48
6	4.938	01	01	02	03	04	05	08	11	20	32	47	63
7	4.718	01	01	02	03	04	06	10	14	26	42	59	75
8	4.568	01	02	02	03	05	08	12	17	32	51	69	84
9	4.460	01	02	02	04	06	10	15	21	39	59	78	90
10	4.378	01	02	03	04	07	11	17	25	45	67	84	94
11	4.313	01	02	03	05	08	13	20	29	52	74	89	97
12	4.262	01	02	03	05	09	15	23	34	58	79	92	98
13	4.219	01	02	03	06	10	17	27	38	63	84	95	99
14	4.183	01	02	04	07	12	19	30	42	68	88	97	99
15	4.153	01	02	04	07	13	22	33	46	73	91	98	*
16	4.126	01	02	04	08	14	24	36	50	77	93	99	
17	4.104	01	02	04	09	16	26	40	54	81	95	99	
18	4.084	01	02	05	09	17	29	43	58	84	96	99	
19	4.067	01	02	05	10	19	31	46	62	86	97	*	
20	4.051	01	03	05	11	20	33	49	65	89	98		
21	4.038	01	03	06	11	22	36	52	68	91	99		
22	4.025	01	03	06	12	23	38	55	71	92	99		
23	4.013	01	03	06	13	25	40	58	74	94	99		
24	4.003	01	03	07	14	26	43	61	77	95	99		
25	3.993	01	03	07	15	28	45	63	79	96	*		
26	3.984	01	03	07	16	30	48	66	81	97			
27	3.976	01	03	08	17	31	50	68	83	97			
28	3.969	02	03	08	18	33	52	71	85	98			
29	3.962	02	04	08	19	35	54	73	87	98			
30	3.955	02	04	09	20	36	56	75	88	99			
31	3.949	02	04	09	21	38	58	77	90	99			
32	3.944	02	04	10	22	40	60	79	91	99			
33	3.939	02	04	10	23	41	62	80	92	99			
34	3.934	02	04	10	24	43	64	82	93	99			
35	3.929	02	04	11	25	45	66	83	94	*			
36	3.925	02	04	11	26	46	68	85	94				
37	3.921	02	05	12	27	48	70	86	95				
38	3.917	02	05	12	28	49	71	87	96				
39	3.914	02	05	13	29	51	73	88	96				

Table 8.3.3 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	3.910	02	05	13	30	53	74	89	97	*	*	*	*
42	3.904	02	05	14	32	56	77	91	98				
44	3.898	02	06	15	34	58	80	93	98				
46	3.893	02	06	16	36	61	82	94	99				
48	3.889	02	06	17	38	64	84	95	99				
50	3.884	02	06	18	41	66	86	96	99				
52	3.880	02	07	19	43	69	88	97	99				
54	3.876	02	07	21	45	71	90	97	*				
56	3.873	02	07	22	47	73	91	98					
58	3.870	02	08	23	49	75	92	98					
60	3.867	02	08	24	51	77	93	99					
64	3.862	02	09	26	55	81	95	99					
68	3.857	02	09	29	59	84	96	99					
72	3.853	03	10	31	62	87	97	*					
76	3.849	03	11	34	65	89	98						
80	3.845	03	11	36	69	91	99						
84	3.842	03	12	38	72	93	99						
88	3.839	03	13	41	74	94	99						
92	3.837	03	14	43	77	95	99						
96	3.834	03	15	45	79	96	*						
100	3.832	03	16	48	81	97							
120	3.824	04	21	59	90	99							
140	3.818	04	26	68	95	*							
160	3.813	05	31	76	97								
180	3.810	06	36	82	99								
200	3.807	07	42	87	99								
250	3.802	09	54	95	*								
300	3.798	11	66	98									
350	3.796	13	75	99									
400	3.794	16	82	*									
450	3.793	19	87										
500	3.792	22	91										
600	3.790	29	96										
700	3.789	35	98										
800	3.788	42	99										
900	3.787	49	*										
1000	3.787	55											

* Power values below this point are greater than .995.

Table 8.3.4

Power of F test at a = .01, u = 4

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	11.392	01	01	01	01	02	02	02	03	04	05	06	08
3	5.994	01	01	01	02	02	02	03	04	06	10	14	20
4	4.893	01	01	01	02	03	03	04	06	11	18	27	39
5	4.431	01	01	02	02	03	05	06	09	17	28	42	57
6	4.177	01	01	02	03	04	06	09	12	23	39	56	73
7	4.018	01	01	02	03	05	08	11	16	31	50	69	84
8	3.910	01	02	02	04	06	09	14	21	39	60	78	91
9	3.828	01	02	03	04	07	11	17	25	46	69	86	95
10	3.769	01	02	03	05	08	13	21	30	54	76	91	97
11	3.721	01	02	03	05	09	15	24	35	60	82	94	99
12	3.682	01	02	03	06	11	18	28	40	67	87	96	99
13	3.649	01	02	04	07	12	20	32	45	72	90	98	*
14	3.623	01	02	04	07	13	23	35	50	77	93	99	
15	3.601	01	02	04	08	15	26	39	54	81	95	99	
16	3.581	01	02	05	09	17	28	43	59	85	97	*	
17	3.564	01	02	05	10	18	31	47	63	88	98		
18	3.549	01	03	05	11	20	34	50	67	90	98		
19	3.536	01	03	06	11	22	37	54	70	92	99		
20	3.524	01	03	06	12	24	39	57	74	94	99		
21	3.514	01	03	06	13	26	42	60	77	95	*		
22	3.504	01	03	07	14	27	45	64	80	96			
23	3.495	01	03	07	15	29	48	67	82	97			
24	3.487	01	03	07	16	31	50	69	84	98			
25	3.480	01	03	08	17	33	53	72	86	98			
26	3.473	01	03	08	19	35	55	74	88	99			
27	3.467	02	04	09	20	37	58	77	90	99			
28	3.462	02	04	09	21	39	60	79	91	99			
29	3.457	02	04	10	22	41	63	81	92	99			
30	3.452	02	04	10	23	43	65	83	93	*			
31	3.448	02	04	11	24	45	67	84	94				
32	3.443	02	04	11	25	47	69	86	95				
33	3.439	02	04	12	27	49	71	87	96				
34	3.436	02	05	12	28	50	73	89	97				
35	3.432	02	05	13	29	52	75	90	97				
36	3.429	02	05	13	30	54	76	91	98				
37	3.426	02	05	14	32	56	78	92	98				
38	3.423	02	05	14	33	57	79	93	98				
39	3.420	02	05	15	34	59	81	94	99				

Table 8.3.4 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	3.418	02	05	15	35	61	82	94	99	*	*	*	*
42	3.413	02	06	17	38	64	85	96	99				
44	3.409	02	06	18	40	67	87	97	99				
46	3.405	02	06	19	43	70	89	97	*				
48	3.401	02	07	20	45	72	91	98					
50	3.398	02	07	22	48	75	92	98					
52	3.395	02	07	23	50	77	93	99					
54	3.392	02	08	24	52	79	94	99					
56	3.389	02	08	26	55	81	95	99					
58	3.386	02	09	27	57	83	96	99					
60	3.384	02	09	28	59	85	97	*					
64	3.380	02	10	31	63	88	98						
68	3.376	03	11	34	67	90	98						
72	3.373	03	11	37	71	92	99						
76	3.371	03	12	39	74	94	99						
80	3.368	03	13	42	77	95	*						
84	3.366	03	14	45	80	96							
88	3.364	03	15	48	82	97							
92	3.361	03	16	50	84	98							
96	3.360	03	17	53	86	98							
100	3.358	03	19	55	88	99							
120	3.352	04	24	67	94	*							
140	3.347	05	30	76	98								
160	3.344	06	37	84	99								
180	3.341	06	43	89	*								
200	3.339	07	49	93									
250	3.335	10	63	98									
300	3.332	12	74	99									
350	3.330	15	82	*									
400	3.329	19	89										
450	3.328	22	93										
500	3.327	26	96										
600	3.326	34	98										
700	3.325	42	*										
800	3.324	49											
900	3.323	56											
1000	3.323	63											

* Power values below this point are greater than .995.

Table 8.3.5

Power of F test at a = .01, u = 5

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	8.746	01	01	01	01	02	02	02	03	04	05	07	09
3	5.064	01	01	01	02	02	03	03	04	07	11	17	24
4	4.248	01	01	02	02	03	04	05	07	12	21	32	46
5	3.895	01	01	02	02	03	05	07	10	19	33	49	66
6	3.699	01	01	02	03	04	07	10	14	28	45	64	80
7	3.576	01	01	02	03	05	08	13	19	36	57	76	90
8	3.489	01	02	02	04	07	10	16	24	45	67	85	95
9	3.426	01	02	03	05	08	13	20	30	53	76	91	98
10	3.388	01	02	03	05	09	15	24	35	61	83	95	99
11	3.339	01	02	03	06	10	18	28	41	68	88	97	*
12	3.309	01	02	04	07	12	21	32	46	74	92	98	
13	3.284	01	02	04	07	14	24	37	52	79	95	99	
14	3.263	01	02	04	08	15	27	41	57	84	97	*	
15	3.244	01	02	05	09	17	30	45	62	87	98		
16	3.229	01	02	05	10	19	33	49	66	90	99		
17	3.215	01	03	05	11	21	36	53	70	92	99		
18	3.203	01	03	06	12	23	39	57	74	94	99		
19	3.192	01	03	06	13	25	42	61	77	96	*		
20	3.182	01	03	07	14	27	45	64	81	97			
21	3.174	01	03	07	15	30	48	68	83	98			
22	3.166	01	03	07	16	32	51	71	86	98			
23	3.159	01	03	08	18	34	54	74	88	99			
24	3.153	01	03	08	19	36	57	76	90	99			
25	3.147	01	04	09	20	38	60	79	91	99			
26	3.142	02	04	09	21	40	63	81	93	*			
27	3.137	02	04	10	23	43	65	83	94				
28	3.133	02	04	10	24	45	67	85	95				
29	3.129	02	04	11	25	47	70	87	96				
30	3.125	02	04	11	27	49	72	88	97				
31	3.121	02	04	12	28	51	74	90	97				
32	3.118	02	05	12	29	53	76	91	98				
33	3.115	02	05	13	31	55	78	92	98				
34	3.112	02	05	14	32	57	80	93	98				
35	3.109	02	05	14	34	59	81	94	99				
36	3.107	02	05	15	35	61	83	95	99				
37	3.104	02	05	16	36	63	84	95	99				
38	3.102	02	06	16	38	64	86	96	99				
39	3.100	02	06	17	39	66	87	97	99				

Table 8.3.5 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	3.097	02	06	18	41	68	88	97	*	*	*	*	*
42	3.093	02	06	19	43	71	90	98					
44	3.090	02	07	20	46	74	92	98					
46	3.087	02	07	22	49	77	93	99					
48	3.084	02	07	23	52	79	94	99					
50	3.081	02	08	25	54	81	96	99					
52	3.079	02	08	26	57	84	96	*					
54	3.076	02	09	28	59	85	97						
56	3.074	02	09	30	61	87	98						
58	3.072	02	10	31	64	89	98						
60	3.070	02	10	33	66	90	99						
64	3.067	03	11	36	70	92	99						
68	3.064	03	12	39	74	94	99						
72	3.061	03	13	42	77	96	*						
76	3.059	03	14	45	80	97							
80	3.057	03	15	48	83	98							
84	3.055	03	16	51	86	98							
88	3.053	03	18	54	88	99							
92	3.052	03	19	57	90	99							
96	3.050	04	20	60	91	99							
100	3.049	04	21	62	93	*							
120	3.044	04	28	74	97								
140	3.040	05	35	83	99								
160	3.037	06	42	89	*								
180	3.035	07	49	93									
200	3.033	08	55	96									
250	3.030	11	70	99									
300	3.028	14	80	*									
350	3.026	18	88										
400	3.025	22	93										
450	3.024	26	96										
500	3.023	30	98										
600	3.022	39	99										
700	3.022	47	*										
800	3.021	56											
900	3.021	63											
1000	3.020	70											

* Power values below this point are greater than .995.

Table 8.3.6

Power of F test at a = .01, u = 6

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	7.191	01	01	01	01	02	02	02	03	04	06	07	10
3	4.456	01	01	01	02	02	03	03	05	08	13	19	28
4	3.812	01	01	02	02	03	04	06	08	14	24	37	53
5	3.528	01	01	02	03	04	06	08	12	22	38	56	73
6	3.369	01	01	02	03	05	07	11	16	32	51	71	86
7	3.266	01	02	02	04	06	09	15	22	41	64	83	94
8	3.196	01	02	03	04	07	12	19	28	51	74	90	97
9	3.143	01	02	03	05	09	14	23	34	60	82	95	99
10	3.103	01	02	03	06	10	17	27	40	68	88	97	*
11	3.072	01	02	03	06	12	20	32	46	74	92	99	
12	3.047	01	02	04	07	13	23	37	52	80	95	99	
13	3.026	01	02	04	08	15	27	41	58	85	97	*	
14	3.008	01	02	05	09	17	30	46	63	89	98		
15	2.992	01	02	05	10	20	34	51	68	92	99		
16	2.979	01	02	05	11	22	37	55	72	94	99		
17	2.968	01	03	06	12	24	41	59	76	95	*		
18	2.957	01	03	06	13	26	44	63	80	97			
19	2.949	01	03	07	15	29	48	67	83	98			
20	2.941	01	03	07	16	31	51	71	86	98			
21	2.934	01	03	08	17	34	54	74	88	99			
22	2.928	01	03	08	19	36	57	77	90	99			
23	2.922	01	03	09	20	38	60	80	92	99			
24	2.917	02	04	09	21	41	63	82	93	*			
25	2.912	02	04	10	23	43	66	84	95				
26	2.908	02	04	10	24	46	69	86	96				
27	2.904	02	04	11	26	48	71	88	96				
28	2.900	02	04	11	27	50	74	90	97				
29	2.896	02	04	12	29	53	76	91	98				
30	2.893	02	05	13	30	55	78	92	98				
31	2.890	02	05	13	32	57	80	93	99				
32	2.887	02	05	14	33	59	82	94	99				
33	2.884	02	05	15	35	61	83	95	99				
34	2.882	02	05	15	36	63	85	96	99				
35	2.880	02	05	16	38	65	86	97	99				
36	2.877	02	06	17	40	67	88	97	*				
37	2.875	02	06	18	41	69	89	98					
38	2.873	02	06	18	43	71	90	98					
39	2.871	02	06	19	44	72	91	98					

Table 8.3.6 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.870	02	06	20	46	74	92	99	*	*	*	*	*
42	2.866	02	07	22	49	77	94	99					
44	2.863	02	07	23	52	80	95	99					
46	2.861	02	08	25	55	82	96	*					
48	2.858	02	08	27	57	85	97						
50	2.856	02	09	28	60	87	98						
52	2.854	02	09	30	63	88	98						
54	2.852	02	10	32	65	90	99						
56	2.850	02	10	33	68	91	99						
58	2.848	02	11	35	70	93	99						
60	2.847	02	11	37	72	94	99						
64	2.844	03	12	40	76	95	*						
68	2.841	03	13	44	80	97							
72	2.839	03	14	47	83	98							
76	2.837	03	16	51	86	98							
80	2.835	03	17	54	88	99							
84	2.834	03	18	57	90	99							
88	2.832	03	20	60	92	99							
92	2.831	04	21	63	93	*							
96	2.830	04	23	66	95								
100	2.829	05	24	69	96								
120	2.825	05	32	80	99								
140	2.821	06	39	88	*								
160	2.819	07	47	93									
180	2.817	08	54	96									
200	2.815	09	61	98									
250	2.813	12	76	*									
300	2.811	16	86										
350	2.810	20	92										
400	2.809	24	96										
450	2.808	29	98										
500	2.807	34	99										
600	2.806	44	*										
700	2.806	53											
800	2.805	62											
900	2.805	69											
1000	2.805	76											

* Power values below this point are greater than .995.

Table 8.3.7

Power of F test at a = .01, u = 8

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	5.467	01	01	01	01	02	02	02	03	05	06	09	12
3	3.705	01	01	01	02	02	03	04	06	10	16	25	37
4	3.256	01	01	02	02	03	05	07	09	18	31	47	65
5	3.053	01	01	02	03	04	06	10	14	28	47	67	84
6	2.936	01	01	02	03	05	09	14	20	40	63	82	93
7	2.861	01	02	02	04	07	11	18	27	51	75	91	98
8	2.808	01	02	03	05	08	14	23	34	61	84	96	99
9	2.770	01	02	03	06	10	18	28	42	71	90	98	*
10	2.740	01	02	03	07	12	21	34	49	78	94	99	
11	2.716	01	02	04	08	14	25	40	56	84	97	*	
12	2.697	01	02	04	09	17	29	45	62	89	98		
13	2.681	01	02	05	10	19	33	51	68	92	99		
14	2.667	01	02	05	11	22	37	56	74	95	*		
15	2.656	01	03	06	12	24	42	61	78	96			
16	2.646	01	03	06	13	27	46	66	82	98			
17	2.638	01	03	07	15	30	50	70	86	98			
18	2.630	01	03	07	16	33	54	74	88	99			
19	2.624	01	03	08	18	35	57	77	91	99			
20	2.618	01	03	08	20	38	61	81	93	*			
21	2.612	01	03	09	21	41	64	83	94				
22	2.608	01	04	10	23	44	68	86	96				
23	2.603	02	04	10	25	47	71	88	97				
24	2.599	02	04	11	26	50	74	90	97				
25	2.596	02	04	12	28	52	76	92	98				
26	2.592	02	04	12	30	55	79	93	98				
27	2.589	02	05	13	32	58	81	94	99				
28	2.586	02	05	14	34	60	83	95	99				
29	2.583	02	05	15	35	63	85	96	99				
30	2.581	02	05	15	37	65	87	97	*				
31	2.579	02	05	16	39	67	88	97					
32	2.576	02	06	17	41	70	90	98					
33	2.574	02	06	18	43	72	91	98					
34	2.573	02	06	19	45	74	92	99					
35	2.571	02	06	20	46	75	93	99					
36	2.569	02	06	21	48	77	94	99					
37	2.567	02	07	22	50	79	95	99					
38	2.566	02	07	23	52	80	95	99					
39	2.564	02	07	24	54	82	96	*					

Table 8.3.7 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.563	02	07	25	55	83	97	*	*	*	*	*	*
42	2.561	02	08	27	58	86	97						
44	2.558	02	08	29	62	88	98						
46	2.556	02	09	31	65	90	99						
48	2.554	02	10	33	68	92	99						
50	2.553	02	10	35	70	93	99						
52	2.551	02	11	37	73	94	99						
54	2.550	02	11	39	75	95	*						
56	2.548	03	12	41	78	96							
58	2.547	03	13	43	80	97							
60	2.546	03	13	45	82	97							
64	2.543	03	15	49	85	98							
68	2.541	03	16	53	88	99							
72	2.540	03	18	57	90	99							
76	2.538	03	20	61	92	*							
80	2.537	03	21	64	94								
84	2.536	04	23	67	95								
88	2.535	04	24	70	96								
92	2.534	04	26	73	97								
96	2.533	04	28	76	98								
100	2.532	04	30	78	98								
120	2.529	05	39	88	*								
140	2.526	06	48	94									
160	2.524	07	57	97									
180	2.523	09	65	99									
200	2.521	10	72	99									
250	2.519	14	85	*									
300	2.518	19	92										
350	2.517	25	97										
400	2.516	30	99										
450	2.516	36	99										
500	2.515	42	*										
600	2.515	53											
700	2.514	63											
800	2.514	72											
900	2.514	79											
1000	2.513	85											

* Power values below this point are greater than .995.

Table 8.3.8

Power of F test at a = .01, u = 10

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	4.539	01	01	01	01	02	02	03	03	05	07	10	15
3	3.258	01	01	02	02	02	03	04	06	11	20	31	46
4	2.914	01	01	02	02	03	05	08	11	22	38	57	74
5	2.752	01	01	02	03	05	07	11	17	34	56	77	91
6	2.662	01	01	02	04	06	10	16	25	47	72	89	97
7	2.603	01	02	03	05	08	13	22	33	60	83	95	99
8	2.561	01	03	03	06	10	17	28	41	70	91	98	*
9	2.530	01	03	03	07	12	21	34	49	79	95	99	
10	2.506	01	03	04	08	14	25	40	57	86	97	*	
11	2.487	01	03	04	09	17	30	47	65	97	99		
12	2.471	01	03	05	10	20	35	53	71	94	99		
13	2.458	01	03	05	11	23	40	59	77	96	*		
14	2.448	01	03	06	13	26	44	65	82	98			
15	2.439	01	03	06	14	29	49	70	86	99			
16	2.431	01	03	07	16	32	53	74	89	99			
17	2.424	01	03	08	18	35	58	78	91	*			
18	2.418	01	03	08	19	39	62	82	94				
19	2.413	01	03	09	21	42	66	85	95				
20	2.408	01	04	10	23	45	69	88	96				
21	2.403	02	04	10	25	49	73	90	97				
22	2.399	02	04	11	27	52	76	92	98				
23	2.396	02	04	12	29	55	79	93	99				
24	2.393	02	04	13	31	58	81	95	99				
25	2.390	02	05	13	33	61	84	96	99				
26	2.387	02	05	14	35	63	86	97	*				
27	2.384	02	05	15	38	66	88	97					
28	2.382	02	05	16	40	69	90	98					
29	2.380	02	05	17	42	71	91	98					
30	2.378	02	06	18	44	73	92	99					
31	2.376	02	06	19	46	76	93	99					
32	2.374	02	06	20	48	78	94	99					
33	2.372	02	06	21	50	80	95	99					
34	2.371	02	07	22	52	81	96	*					
35	2.369	02	07	24	54	83	97						
36	2.368	02	07	25	56	85	97						
37	2.367	02	08	26	58	86	98						
38	2.365	02	08	27	60	87	98						
39	2.364	02	08	28	62	89	98						

Table 8.3.8 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.363	02	08	29	63	90	99	*	*	*	*	*	*
42	2.361	02	09	32	67	92	99						
44	2.359	02	10	34	70	93	99						
46	2.358	02	10	36	73	95	*						
48	2.356	02	11	39	76	96							
50	2.355	02	12	41	78	97							
52	2.353	03	12	43	81	97							
54	2.352	03	13	46	83	98							
56	2.351	03	14	48	85	98							
58	2.350	03	15	50	87	99							
60	2.349	03	16	53	88	99							
64	2.347	03	17	57	91	99							
68	2.346	03	19	61	93	*							
72	2.344	03	21	65	95								
76	2.343	04	23	69	96								
80	2.342	04	25	72	97								
84	2.341	04	27	75	98								
88	2.340	04	29	78	99								
92	2.339	04	31	81	99								
96	2.338	05	33	83	99								
100	2.338	05	35	86	99								
120	2.335	06	46	93	*								
140	2.333	07	56	97									
160	2.331	08	65	99									
180	2.330	10	73	*									
200	2.329	12	79										
250	2.327	17	91										
300	2.326	23	96										
350	2.326	29	99										
400	2.325	36	*										
450	2.325	42											
500	2.324	51											
600	2.324	61											
700	2.323	71											
800	2.323	80											
900	2.323	86											
1000	2.323	91											

* Power values below this point are greater than .995.

Table 8.3.9

Power of F test at a = .01, u = 12

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	3.960	01	01	01	01	02	02	03	04	05	08	12	18
3	2.958	01	01	01	02	03	04	05	07	13	23	37	54
4	2.679	01	01	02	03	04	06	09	13	26	44	65	82
5	2.548	01	01	02	03	05	08	13	20	40	64	84	95
6	2.472	01	02	02	04	07	12	19	29	54	79	94	99
7	2.422	01	02	03	05	09	15	25	38	67	89	98	*
8	2.387	01	02	03	06	11	20	32	48	78	95	99	
9	2.361	01	02	04	07	14	25	39	57	85	98	*	
10	2.340	01	02	04	08	17	30	47	65	91	99		
11	2.325	01	02	05	10	20	35	54	72	94	*		
12	2.312	01	02	05	11	23	40	60	78	97			
13	2.301	01	03	06	13	26	45	66	83	98			
14	2.292	01	03	06	15	30	51	72	87	99			
15	2.285	01	03	07	16	33	56	77	91	99			
16	2.278	01	03	08	18	37	60	81	93	*			
17	2.272	01	03	08	20	41	65	84	95				
18	2.267	01	03	09	23	45	69	87	97				
19	2.262	01	04	10	25	48	73	90	98				
20	2.258	02	04	11	27	52	76	92	98				
21	2.255	02	04	12	29	55	80	94	99				
22	2.251	02	04	13	32	59	83	95	99				
23	2.248	02	05	14	34	62	85	96	99				
24	2.246	02	05	15	36	65	87	97	*				
25	2.243	02	05	16	39	68	89	98					
26	2.241	02	05	17	41	71	91	98					
27	2.239	02	05	18	43	73	92	99					
28	2.237	02	06	19	46	76	94	99					
29	2.235	02	06	20	48	78	95	99					
30	2.233	02	06	21	50	80	96	*					
31	2.231	02	07	22	53	82	96						
32	2.230	02	07	24	55	84	97						
33	2.228	02	07	25	57	86	98						
34	2.227	02	07	26	59	87	98						
35	2.226	02	08	27	61	88	98						
36	2.225	02	08	29	63	90	99						
37	2.224	02	08	30	65	91	99						
38	2.223	02	09	31	67	92	99						
39	2.222	02	09	32	69	93	99						

Table 8.3.9 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.221	02	09	34	71	94	99	*	*	*	*	*	*
42	2.219	02	10	36	74	95	*						
44	2.217	02	11	39	77	96							
46	2.216	02	12	42	80	97							
48	2.215	02	12	44	82	98							
50	2.213	03	13	47	85	98							
52	2.212	03	14	50	87	99							
54	2.211	03	15	52	88	99							
56	2.210	03	16	55	90	99							
58	2.209	03	17	57	91	*							
60	2.209	03	18	59	93								
64	2.207	03	20	64	95								
68	2.206	03	22	68	96								
72	2.204	04	24	72	97								
76	2.203	04	26	76	98								
80	2.202	04	29	79	99								
84	2.202	04	31	82	99								
88	2.201	04	33	84	99								
92	2.200	05	36	87	*								
96	2.199	05	38	89									
100	2.199	05	40	91									
120	2.197	07	52	96									
140	2.195	08	63	99									
160	2.194	10	72	*									
180	2.193	12	79										
200	2.192	14	85										
250	2.191	20	94										
300	2.190	26	98										
350	2.189	34	99										
400	2.188	41	*										
450	2.188	48											
500	2.188	55											
600	2.187	68											
700	2.187	78											
800	2.187	86											
900	2.186	91											
1000	2.186	94											

* Power values below this point are greater than .995.

Table 8.3.10

Power of F test at a = .01, u = 15

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	3.409	01	01	01	01	02	02	03	04	06	10	15	23
3	2.656	01	01	02	02	03	04	06	08	16	29	46	64
4	2.437	01	01	02	03	04	07	10	15	31	53	75	90
5	2.332	01	01	02	04	06	10	16	25	48	74	91	98
6	2.272	01	02	03	05	08	14	23	35	64	87	97	*
7	2.232	01	02	03	06	10	19	31	46	77	94	99	
8	2.203	01	02	03	07	13	24	39	56	86	98	*	
9	2.182	01	02	04	08	16	30	47	66	92	99		
10	2.166	01	02	05	10	20	36	55	74	95	*		
11	2.153	01	02	05	11	24	42	63	81	98			
12	2.143	01	03	06	13	28	48	69	86	99			
13	2.134	01	03	07	15	32	54	75	90	99			
14	2.127	01	03	07	17	36	59	80	93	*			
15	2.120	01	03	08	20	40	65	85	95				
16	2.115	01	03	09	22	44	69	88	97				
17	2.110	01	04	10	25	49	74	91	98				
18	2.106	01	04	11	27	53	78	93	99				
19	2.102	02	04	12	30	57	81	95	99				
20	2.099	02	04	13	32	60	84	96	99				
21	2.096	02	04	14	35	64	87	97	*				
22	2.093	02	05	15	38	68	89	98					
23	2.091	02	05	16	41	71	91	99					
24	2.088	02	05	17	43	74	93	99					
25	2.086	02	06	19	46	77	94	99					
26	2.084	02	06	20	49	79	95	*					
27	2.083	02	06	21	51	81	96						
28	2.081	02	07	23	54	84	97						
29	2.079	02	07	24	56	86	98						
30	2.078	02	07	25	59	87	98						
31	2.077	02	08	27	61	89	99						
32	2.076	02	08	28	63	90	99						
33	2.074	02	08	30	66	92	99						
34	2.073	02	09	31	68	93	99						
35	2.072	02	09	33	70	94	99						
36	2.071	02	09	34	72	95	*						
37	2.070	02	10	36	74	95							
38	2.070	02	10	37	76	96							
39	2.069	02	11	39	77	97							

Table 8.3.10 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.068	02	11	40	79	97	*	*	*	*	*	*	*
42	2.066	02	12	43	82	98							
44	2.065	02	13	46	85	99							
46	2.064	03	14	49	87	99							
48	2.063	03	15	52	89	99							
50	2.062	03	16	55	91	99							
52	2.061	03	17	58	92	*							
54	2.060	03	18	61	94								
56	2.059	03	19	63	95								
58	2.059	03	20	66	96								
60	2.058	03	22	68	96								
64	2.057	03	24	73	98								
68	2.056	04	26	77	98								
72	2.055	04	29	80	99								
76	2.054	04	32	84	99								
80	2.053	04	34	86	*								
84	2.052	05	37	89									
88	2.052	05	40	91									
92	2.051	05	43	92									
96	2.051	06	45	94									
100	2.050	06	48	95									
120	2.048	07	61	98									
140	2.047	09	71	*									
160	2.046	11	80										
180	2.045	14	87										
200	2.044	16	91										
250	2.043	24	97										
300	2.042	32	99										
350	2.042	40	*										
400	2.041	48											
450	2.041	57											
500	2.041	64											
600	2.040	76											
700	2.040	86											
800	2.040	92											
900	2.040	95											
1000	2.040	98											

* Power values below this point are greater than .995.

Table 8.3.11

Power of F test at a = .01, u = 24

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.620	01	01	01	02	02	03	04	05	09	15	25	38
3	2.184	01	01	02	02	04	05	08	12	26	46	68	85
4	2.049	01	01	02	03	06	09	15	24	48	75	92	98
5	1.983	01	02	03	05	08	15	24	38	69	91	99	*
6	1.944	01	02	03	06	11	21	35	52	84	97	*	
7	1.918	01	02	04	08	15	29	46	66	92	99		
8	1.900	01	02	04	10	20	37	57	76	97	*		
9	1.886	01	02	05	12	25	45	67	85	99			
10	1.876	01	03	06	14	30	53	75	90	*			
11	1.867	02	03	07	17	36	60	82	94				
12	1.860	02	03	08	20	41	67	87	96				
13	1.854	02	03	09	23	47	73	91	98				
14	1.850	02	04	10	26	53	79	94	99				
15	1.846	02	04	11	30	58	83	96	99				
16	1.842	02	04	13	33	63	87	97	*				
17	1.839	02	04	14	37	68	90	98					
18	1.836	02	05	16	41	72	93	99					
19	1.833	02	05	17	44	76	95	99					
20	1.831	02	05	19	48	80	96	*					
21	1.829	02	06	20	51	83	97						
22	1.827	02	06	22	55	86	98						
23	1.826	02	07	24	58	88	99						
24	1.824	02	07	26	62	90	99						
25	1.823	02	07	28	65	92	99						
26	1.821	02	08	30	68	93	99						
27	1.820	02	08	32	71	95	*						
28	1.819	02	09	34	73	96							
29	1.818	02	09	36	76	96							
30	1.817	02	10	38	78	97							
31	1.816	02	10	40	80	98							
32	1.815	02	11	42	82	98							
33	1.815	02	12	44	84	99							
34	1.814	02	12	46	86	99							
35	1.813	02	13	48	87	99							
36	1.813	02	13	50	89	99							
37	1.812	03	14	52	90	99							
38	1.811	03	15	54	91	*							
39	1.811	03	15	56	92								

Table 8.3.11 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.810	03	16	58	93	*	*	*	*	*	*	*	*
42	1.809	03	17	62	95								
44	1.809	03	19	65	96								
46	1.808	03	20	68	97								
48	1.807	03	22	72	98								
50	1.806	03	24	74	98								
52	1.806	03	25	77	99								
54	1.805	04	27	80	99								
56	1.805	04	29	82	99								
58	1.804	04	30	84	*								
60	1.804	04	32	86									
64	1.803	04	36	89									
68	1.802	05	39	92									
72	1.802	05	43	94									
76	1.801	05	47	95									
80	1.800	06	50	97									
84	1.800	06	54	98									
88	1.800	06	57	98									
92	1.799	07	60	99									
96	1.799	07	64	99									
100	1.799	08	67	99									
120	1.797	10	79	*									
140	1.796	13	88										
160	1.796	16	94										
180	1.795	20	97										
200	1.795	24	98										
250	1.794	35	*										
300	1.793	46											
350	1.793	57											
400	1.793	67											
450	1.793	75											
500	1.792	82											
600	1.792	92											
700	1.792	96											
800	1.792	99											
900	1.792	99											
1000	1.792	*											

* Power values below this point are greater than .995.

Table 8.3.12

Power of F test at a = .05, u = 1

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	18.513	05	05	06	06	07	07	08	09	10	12	14	16
3	7.709	05	05	06	07	08	09	10	12	16	20	26	32
4	5.987	05	06	06	07	09	11	13	16	23	30	39	48
5	5.318	05	06	07	08	11	13	16	20	29	39	50	61
6	4.965	05	06	07	09	12	15	20	24	35	47	60	71
7	4.747	05	06	08	10	14	18	23	28	41	55	68	79
8	4.600	05	06	08	11	15	20	26	32	47	62	75	85
9	4.494	05	07	09	12	17	22	29	36	52	68	80	89
10	4.414	05	07	09	13	18	25	32	40	57	73	85	93
11	4.351	05	07	10	14	20	27	35	44	62	77	88	95
12	4.301	05	07	10	15	22	29	38	47	66	81	91	97
13	4.260	05	07	11	16	23	32	41	51	70	84	93	98
14	4.225	05	08	11	17	25	34	44	54	73	87	95	98
15	4.196	06	08	12	18	26	36	47	57	76	89	96	99
16	4.171	06	08	12	19	28	38	49	60	79	91	97	99
17	4.149	06	08	13	20	30	40	52	63	82	93	98	*
18	4.130	06	08	14	21	31	42	54	66	84	94	98	
19	4.113	06	09	14	22	33	44	57	68	86	95	99	
20	4.098	06	09	15	23	34	46	59	70	88	96	99	
21	4.085	06	09	15	24	36	48	61	73	89	97	99	
22	4.073	06	09	16	26	37	50	63	75	91	97	*	
23	4.062	06	10	16	27	39	52	65	77	92	98		
24	4.052	06	10	17	28	40	54	67	78	93	98		
25	4.043	06	10	18	29	42	56	69	80	94	99		
26	4.034	06	10	18	30	43	58	71	82	95	99		
27	4.026	06	10	19	31	45	59	72	83	95	99		
28	4.020	06	11	19	32	46	61	74	84	96	99		
29	4.013	06	11	20	33	47	62	76	86	97	99		
30	4.007	06	11	21	34	49	64	77	87	97	*		
31	4.001	06	11	21	35	50	65	78	88	97			
32	3.996	06	12	22	36	51	67	80	89	98			
33	3.991	06	12	22	37	53	68	81	90	98			
34	3.986	07	12	23	38	54	69	82	91	98			
35	3.982	07	12	24	39	55	71	83	92	99			
36	3.978	07	13	24	40	56	72	84	92	99			
37	3.974	07	13	25	40	58	73	85	93	99			
38	3.970	07	13	25	41	59	74	86	94	99			
39	3.967	07	13	26	42	60	75	87	94	99			

Table 8.3.12 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	3.963	07	14	27	43	61	77	88	95	99	*	*	*
42	3.957	07	14	28	45	63	79	89	96	*			
44	3.952	07	15	29	47	65	80	91	96				
46	3.947	07	15	30	49	67	82	92	97				
48	3.942	07	16	31	50	69	84	93	97				
50	3.938	07	16	32	52	71	85	94	98				
52	3.934	08	17	33	53	73	87	95	98				
54	3.931	08	17	34	55	74	88	95	99				
56	3.928	08	18	36	57	76	89	96	99				
58	3.924	08	18	37	58	77	90	97	99				
60	3.922	08	19	38	60	79	91	97	99				
64	3.916	08	20	40	62	81	93	98	*				
68	3.912	08	21	42	65	83	94	98					
72	3.908	09	22	44	68	85	95	99					
76	3.904	09	23	46	70	87	96	99					
80	3.901	09	24	48	72	89	97	99					
84	3.898	09	25	50	74	90	97	*					
88	3.895	09	26	52	76	92	98						
92	3.893	10	27	54	78	93	98						
96	3.891	10	28	55	80	94	99						
100	3.889	10	29	57	81	94	99						
120	3.881	11	34	65	88	97	*						
140	3.875	13	39	72	92	99							
160	3.871	14	44	77	95	99							
180	3.868	15	48	82	97	*							
200	3.865	16	52	86	98								
250	3.860	20	62	92	99								
300	3.857	23	70	96	*								
350	3.855	26	76	98									
400	3.853	30	82	99									
450	3.852	33	86	*									
500	3.851	36	89										
600	3.849	42	94										
700	3.848	47	97										
800	3.847	53	98										
900	3.847	58	99										
1000	3.846	62	99										

* Power values below this point are greater than .995.

Table 8.3.13

Power of F test at a = .05, u = 2

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	9.552	05	05	06	06	07	07	08	08	10	12	15	18
3	5.143	05	05	06	07	08	09	10	12	17	22	29	37
4	4.256	05	06	06	08	09	11	14	17	24	33	44	54
5	3.885	05	06	07	09	11	14	17	22	32	44	56	69
6	3.682	05	06	07	10	13	16	21	26	39	53	67	79
7	3.555	05	06	08	11	14	19	25	31	46	62	76	87
8	3.467	05	06	08	12	16	22	28	36	53	69	83	92
9	3.403	05	07	09	13	18	24	32	40	59	75	88	95
10	3.354	05	07	10	14	20	27	35	45	64	81	91	97
11	3.316	05	07	10	15	21	30	39	49	69	85	94	98
12	3.285	06	07	11	16	23	32	42	53	74	88	96	99
13	3.260	06	08	11	17	25	35	46	57	77	91	97	99
14	3.238	06	08	12	18	27	38	49	61	81	93	98	*
15	3.220	06	08	13	20	29	40	52	64	84	95	99	
16	3.205	06	08	13	21	31	43	55	67	86	96	99	
17	3.191	06	09	14	22	33	45	58	70	89	97	99	
18	3.179	06	09	14	23	34	48	61	73	90	98	*	
19	3.168	06	09	15	24	36	50	64	76	92	99		
20	3.159	06	09	16	26	38	52	66	78	93	99		
21	3.150	06	09	16	27	40	54	69	80	95	99		
22	3.143	06	10	17	28	42	57	71	82	96	99		
23	3.136	06	10	18	29	43	59	73	84	96	*		
24	3.130	06	10	18	30	45	61	75	86	97			
25	3.124	06	10	19	32	47	63	77	87	98			
26	3.119	06	11	20	33	48	65	79	89	98			
27	3.114	06	11	20	34	50	66	80	90	98			
28	3.110	06	11	21	35	52	68	82	91	99			
29	3.105	06	12	22	36	53	70	83	92	99			
30	3.102	06	12	22	37	55	71	85	93	99			
31	3.098	07	12	23	39	56	73	86	94	99			
32	3.095	07	12	24	40	58	75	87	94	99			
33	3.091	07	13	24	41	59	76	88	95	*			
34	3.088	07	13	25	42	61	77	89	96				
35	3.086	07	13	26	43	62	79	90	96				
36	3.083	07	13	26	44	63	80	91	97				
37	3.081	07	14	27	45	65	81	92	97				
38	3.078	07	14	28	46	66	82	92	97				
39	3.076	07	14	28	47	67	83	93	98				

Table 8.3.13 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	3.074	07	15	29	48	68	84	94	98	*	*	*	*
42	3.070	07	15	30	51	71	86	95	98				
44	3.066	07	16	32	53	73	88	96	99				
46	3.063	07	16	33	55	75	89	96	99				
48	3.060	08	17	34	57	77	90	97	99				
50	3.058	08	18	36	58	79	92	98	99				
52	3.055	08	18	37	60	80	93	98	*				
54	3.053	08	19	38	62	82	94	98					
56	3.051	08	19	40	64	83	94	99					
58	3.049	08	20	41	65	85	95	99					
60	3.047	08	21	42	67	86	96	99					
64	3.044	08	22	45	70	88	97	99					
68	3.041	09	23	47	73	90	98	*					
72	3.039	09	24	49	75	92	98						
76	3.036	09	25	52	78	93	99						
80	3.034	09	27	54	80	94	99						
84	3.032	10	28	56	82	95	99						
88	3.031	10	29	58	84	96	99						
92	3.029	10	30	60	85	97	*						
96	3.028	10	31	62	87	97							
100	3.026	11	32	64	88	98							
120	3.021	12	38	73	94	99							
140	3.018	14	44	79	97	*							
160	3.015	15	49	85	98								
180	3.013	16	54	89	99								
200	3.011	18	59	92	*								
250	3.008	22	69	97									
300	3.006	25	78	99									
350	3.004	29	84	*									
400	3.003	33	89										
450	3.002	36	92										
500	3.002	40	95										
600	3.001	47	98										
700	3.000	53	99										
800	3.000	59	*										
900	2.999	65											
1000	2.999	70											

* Power values below this point are greater than .995.

Table 8.3.14

Power of F test at a = .05, u = 3

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	6.591	05	05	06	06	07	07	08	09	11	13	17	20
3	4.066	05	05	06	07	08	09	11	13	18	25	33	42
4	3.490	05	06	07	08	10	12	15	18	27	38	50	62
5	3.239	05	06	07	09	12	15	19	24	36	50	64	76
6	3.098	05	06	08	10	13	18	23	29	44	60	75	86
7	3.009	05	06	08	11	15	21	27	35	52	69	83	92
8	2.947	05	07	09	12	17	24	31	40	59	77	89	96
9	2.901	05	07	09	14	19	27	36	46	66	82	93	98
10	2.867	05	07	10	15	21	30	40	51	71	87	96	99
11	2.839	06	07	11	16	24	33	44	55	76	91	97	99
12	2.817	06	08	11	17	26	36	48	60	81	93	98	*
13	2.798	06	08	12	19	28	39	52	64	84	95	99	
14	2.783	06	08	13	20	30	42	55	68	87	97	99	
15	2.770	06	08	13	21	32	45	59	71	90	98	*	
16	2.758	06	09	14	23	34	48	62	75	92	98		
17	2.748	06	09	15	24	37	51	65	78	94	99		
18	2.740	06	09	16	26	39	53	68	80	95	99		
19	2.732	06	09	16	27	41	56	71	83	96	99		
20	2.725	06	10	17	28	43	59	73	85	97	*		
21	2.719	06	10	18	30	45	61	76	87	98			
22	2.714	06	10	18	31	47	63	78	88	98			
23	2.709	06	10	19	32	49	66	80	90	99			
24	2.704	06	11	20	34	51	68	82	91	99			
25	2.700	06	11	21	35	53	70	84	93	99			
26	2.696	06	11	22	37	54	72	85	94	99			
27	2.692	07	12	22	38	56	74	87	94	99			
28	2.689	07	12	23	39	58	75	88	95	*			
29	2.686	07	12	24	41	60	77	89	96				
30	2.683	07	13	25	42	61	79	90	96				
31	2.680	07	13	25	43	63	80	91	97				
32	2.678	07	13	26	45	65	81	92	97				
33	2.675	07	14	27	46	66	83	93	98				
34	2.673	07	14	28	47	68	54	94	98				
35	2.671	07	14	29	48	69	85	94	98				
36	2.669	07	14	29	50	70	86	95	99				
37	2.668	07	15	30	51	72	87	96	99				
38	2.666	07	15	31	52	73	88	96	99				
39	2.664	07	15	32	53	74	89	97	99				

Table 8.3.14 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.663	07	16	32	54	76	90	97	99	*	*	*	*
42	2.660	07	16	34	57	78	91	98	*				
44	2.657	08	17	35	59	80	93	98					
46	2.655	08	18	37	61	82	94	99					
48	2.653	08	18	39	63	84	95	99					
50	2.651	08	19	40	65	85	96	99					
52	2.649	08	20	42	67	87	96	99					
54	2.648	08	20	43	69	88	97	99					
56	2.646	08	21	45	71	89	97	*					
58	2.645	08	22	46	72	90	98						
60	2.643	09	22	47	74	91	98						
64	2.641	09	24	50	77	93	99						
68	2.639	09	25	53	80	95	99						
72	2.637	09	27	56	82	96	99						
76	2.635	10	28	58	84	97	*						
80	2.633	10	29	61	86	97							
84	2.632	10	31	63	88	98							
88	2.631	10	32	65	90	98							
92	2.630	11	34	67	91	99							
96	2.629	11	35	69	92	99							
100	2.628	11	36	71	93	99							
120	2.624	13	43	80	97	*							
140	2.621	14	49	86	99								
160	2.619	16	55	91	99								
180	2.618	18	61	94	*								
200	2.616	19	66	96									
250	2.614	24	77	99									
300	2.612	28	84	*									
350	2.611	32	90										
400	2.611	37	93										
450	2.610	41	96										
500	2.609	45	98										
600	2.609	53	99										
700	2.608	60	*										
800	2.608	66											
900	2.607	72											
1000	2.607	77											

* Power values below this point are greater than .995.

Table 8.3.15

Power of F test at a = .05, u = 4

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	5.192	05	05	06	07	08	08	09	10	13	15	19	24
3	3.478	05	05	06	07	09	10	12	14	20	28	38	48
4	3.056	05	06	07	08	10	13	16	20	30	42	56	69
5	2.866	05	06	07	09	12	16	21	26	40	55	70	83
6	2.759	05	06	08	10	14	19	25	32	49	66	81	91
7	2.690	05	06	09	12	16	22	30	39	58	76	88	96
8	2.642	05	07	09	13	19	26	35	45	65	83	93	98
9	2.606	05	07	10	14	21	29	40	51	72	88	96	99
10	2.579	06	07	10	16	23	33	44	56	78	92	98	*
11	2.558	06	08	11	17	26	37	49	61	82	94	99	
12	2.540	06	08	12	19	28	40	53	66	86	96	99	
13	2.525	06	08	13	20	31	43	57	70	89	98	*	
14	2.513	06	08	13	22	33	47	61	74	92	98		
15	2.503	06	09	14	23	36	50	65	78	94	99		
16	2.494	06	09	15	25	38	53	68	81	95	99		
17	2.486	06	09	16	26	40	56	71	83	96	*		
18	2.479	06	09	17	28	43	59	74	86	97			
19	2.473	06	10	17	30	45	62	77	88	98			
20	2.468	06	10	18	31	47	65	79	90	99			
21	2.463	06	10	19	33	50	67	82	91	99			
22	2.458	06	11	20	34	52	69	84	93	99			
23	2.454	06	11	21	36	54	72	85	94	99			
24	2.451	06	11	22	37	56	74	87	95	*			
25	2.447	06	12	23	39	58	76	89	96				
26	2.444	07	12	23	40	60	78	90	96				
27	2.441	07	12	24	42	62	80	91	97				
28	2.439	07	13	25	43	64	81	92	98				
29	2.436	07	13	26	45	66	83	93	98				
30	2.434	07	13	27	46	67	84	94	98				
31	2.432	07	14	28	48	69	86	95	99				
32	2.430	07	14	29	49	71	87	96	99				
33	2.428	07	14	30	51	72	88	96	99				
34	2.427	07	15	30	52	74	89	97	99				
35	2.425	07	15	31	54	75	90	97	99				
36	2.424	07	15	32	55	76	91	97	*				
37	2.422	07	16	33	56	78	92	98					
38	2.421	07	16	34	57	79	92	98					
39	2.419	07	16	35	59	80	93	98					

Table 8.3.15 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.418	07	17	36	60	81	94	99	*	*	*	*	*
42	2.416	08	18	37	62	83	95	99					
44	2.414	08	18	39	65	85	96	99					
46	2.412	08	19	41	67	87	97	99					
48	2.410	08	20	43	69	89	97	*					
50	2.409	08	21	44	71	90	98						
52	2.407	08	21	46	73	91	98						
54	2.406	08	22	48	75	92	99						
56	2.405	09	23	49	77	93	99						
58	2.404	09	24	51	78	94	99						
60	2.403	09	24	52	80	95	99						
64	2.401	09	26	55	83	96	*						
68	2.399	09	28	58	85	97							
72	2.397	10	29	61	87	98							
76	2.396	10	31	64	89	98							
80	2.395	10	32	66	91	99							
84	2.394	11	34	69	92	99							
88	2.393	11	35	71	94	99							
92	2.392	11	37	73	95	*							
96	2.391	11	39	75	96								
100	2.390	12	40	77	96								
120	2.387	13	47	85	99								
140	2.385	15	54	91	99								
160	2.383	17	61	94	*								
180	2.382	18	67	97									
200	2.381	20	72	98									
250	2.379	25	82	*									
300	2.378	29	89										
350	2.377	34	94										
400	2.376	39	96										
450	2.376	44	98										
500	2.376	49	99										
600	2.375	57	*										
700	2.374	65											
800	2.374	72											
900	2.374	78											
1000	2.374	82											

* Power values below this point are greater than .995.

Table 8.3.16

Power of F test at a = .05, u = 5

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	4.387	05	05	06	07	08	08	09	10	13	17	21	26
3	3.106	05	06	06	07	09	11	13	15	22	31	42	53
4	2.773	05	06	07	08	11	14	17	22	33	47	61	75
5	2.621	05	06	07	10	13	17	22	29	44	61	76	88
6	2.534	05	06	08	11	15	21	27	35	54	72	86	94
7	2.478	05	07	09	12	18	24	33	42	63	81	92	98
8	2.438	05	07	09	14	20	28	38	49	71	87	96	99
9	2.409	05	07	10	15	23	32	43	55	77	92	98	*
10	2.391	06	07	11	17	25	36	48	61	83	95	99	
11	2.368	06	08	12	19	28	40	53	66	87	97	99	
12	2.354	06	08	13	20	31	44	58	71	90	98	*	
13	2.342	06	08	13	22	33	47	62	75	93	99		
14	2.332	06	09	14	24	36	51	66	79	95	99		
15	2.324	06	09	15	25	39	55	70	82	96	*		
16	2.316	06	09	16	27	42	58	73	85	97			
17	2.310	06	10	17	29	44	61	76	88	98			
18	2.304	06	10	18	30	47	64	79	90	99			
19	2.299	06	10	19	32	49	67	82	92	99			
20	2.294	06	11	20	34	52	70	84	93	99			
21	2.290	06	11	21	36	54	72	86	94	*			
22	2.286	06	11	22	37	57	75	88	95				
23	2.283	06	11	22	39	59	77	90	96				
24	2.280	06	12	23	41	61	79	91	97				
25	2.277	07	12	24	43	63	81	92	98				
26	2.275	07	13	25	44	65	83	93	98				
27	2.272	07	13	26	46	67	84	94	98				
28	2.270	07	13	27	47	69	86	95	99				
29	2.268	07	14	28	49	71	87	96	99				
30	2.266	07	14	29	51	73	88	96	99				
31	2.265	07	14	30	52	74	90	97	99				
32	2.263	07	15	31	54	76	91	97	*				
33	2.262	07	15	32	55	77	92	98					
34	2.260	07	16	33	57	79	93	98					
35	2.259	07	16	34	58	80	93	98					
36	2.257	07	16	35	60	81	94	99					
37	2.256	07	17	36	61	83	95	99					
38	2.255	07	17	37	62	84	95	99					
39	2.254	08	18	38	64	85	96	99					

Table 8.3.16 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.253	08	18	39	65	86	96	99	*	*	*	*	*
42	2.251	08	19	41	68	88	97	*					
44	2.249	08	20	43	70	89	98						
46	2.248	08	21	45	72	91	98						
48	2.246	08	21	47	74	92	99						
50	2.245	08	22	48	76	93	99						
52	2.244	09	23	50	78	94	99						
54	2.243	09	24	52	80	95	99						
56	2.242	09	25	54	82	96	*						
58	2.241	09	26	55	83	96							
60	2.240	09	26	57	85	97							
64	2.238	09	28	60	87	98							
68	2.237	10	30	63	89	99							
72	2.235	10	32	66	91	99							
76	2.234	10	33	69	93	99							
80	2.233	11	35	72	94	99							
84	2.232	11	37	74	95	*							
88	2.232	11	39	76	96								
92	2.231	12	40	78	97								
96	2.230	12	42	80	97								
100	2.229	12	44	82	98								
120	2.227	14	52	89	99								
140	2.225	16	59	94	*								
160	2.224	18	66	97									
180	2.223	20	72	98									
200	2.222	23	77	99									
250	2.220	28	87	*									
300	2.219	33	93										
350	2.218	39	96										
400	2.218	44	98										
450	2.217	49	99										
500	2.217	54	*										
600	2.217	63											
700	2.216	71											
800	2.216	77											
900	2.216	83											
1000	2.216	87											

* Power values below this point are greater than .995.

Table 8.3.17

Power of F test at a = .05, u = 6

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	3.866	05	05	06	07	08	08	09	11	14	18	23	29
3	2.848	05	06	06	08	09	11	13	16	24	34	46	51
4	2.573	05	06	07	09	11	14	18	23	36	51	66	80
5	2.445	05	06	08	10	13	18	24	31	48	66	81	91
6	2.372	05	06	08	11	16	22	30	38	58	77	90	96
7	2.324	05	07	09	13	19	26	35	46	68	85	95	99
8	2.291	05	07	10	15	21	30	41	53	76	91	98	*
9	2.266	06	07	11	16	24	35	47	60	82	94	99	
10	2.246	06	08	11	18	27	39	52	66	87	97	*	
11	2.231	06	08	12	20	30	43	57	71	90	98		
12	2.219	06	08	13	22	33	47	62	76	93	99		
13	2.209	06	09	14	23	36	51	67	80	95	99		
14	2.200	06	09	15	25	39	55	71	83	97	*		
15	2.193	06	09	16	27	42	59	74	86	98			
16	2.186	06	10	17	29	45	62	78	89	98			
17	2.181	06	10	18	31	48	66	81	91	99			
18	2.176	06	10	19	33	51	69	83	93	99			
19	2.171	06	11	20	35	53	72	86	94	*			
20	2.168	06	11	21	37	56	74	88	95				
21	2.164	06	11	22	39	58	77	90	96				
22	2.161	06	12	23	40	61	79	91	97				
23	2.158	07	12	24	42	63	81	93	98				
24	2.156	07	12	25	44	65	83	94	98				
25	2.153	07	13	26	46	68	85	95	99				
26	2.151	07	13	27	48	70	87	96	99				
27	2.149	07	14	28	50	72	88	96	99				
28	2.147	07	14	29	51	74	89	97	99				
29	2.145	07	14	30	53	75	91	97	*				
30	2.144	07	15	31	55	77	92	98					
31	2.142	07	15	33	56	79	93	98					
32	2.141	07	16	34	58	80	93	99					
33	2.140	07	16	35	60	82	94	99					
34	2.138	07	17	36	61	83	95	99					
35	2.137	07	17	37	63	84	96	99					
36	2.136	07	17	38	64	85	96	99					
37	2.135	08	18	39	66	87	97	99					
38	2.134	08	18	40	67	88	97	*					
39	2.133	08	19	41	68	89	97						

Table 8.3.17 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.132	08	19	42	70	89	98	*	*	*	*	*	*
42	2.131	08	20	44	72	91	98						
44	2.129	08	21	46	75	92	99						
46	2.128	08	22	48	77	94	99						
48	2.126	08	23	50	79	95	99						
50	2.125	09	24	52	81	96	99						
52	2.124	09	25	54	82	96	*						
54	2.123	09	26	56	84	97							
56	2.122	09	27	58	86	97							
58	2.122	09	27	60	87	98							
60	2.121	09	28	61	88	98							
64	2.119	10	30	65	91	99							
68	2.118	10	32	68	92	99							
72	2.117	10	34	71	94	99							
76	2.116	11	36	74	95	*							
80	2.115	11	38	76	96								
84	2.114	12	40	78	97								
88	2.114	12	42	81	98								
92	2.113	12	44	83	98								
96	2.112	13	45	84	99								
100	2.112	13	47	86	99								
120	2.110	15	56	92	*								
140	2.108	17	64	96									
160	2.107	19	71	98									
180	2.106	21	76	99									
200	2.105	23	81	*									
250	2.104	29	90										
300	2.103	35	95										
350	2.102	40	98										
400	2.102	46	99										
450	2.102	52	*										
500	2.101	57											
600	2.101	67											
700	2.100	75											
800	2.100	82											
900	2.100	87											
1000	2.100	91											

* Power values below this point are greater than .995.

Table 8.3.18

Power of F test at a = .05, u = 8

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	3.230	05	05	06	07	08	09	10	11	15	20	26	34
3	2.510	05	06	06	08	10	12	15	18	28	40	53	67
4	2.305	05	06	07	09	12	16	21	27	42	59	75	87
5	2.208	05	06	08	11	15	20	27	35	55	74	88	96
6	2.152	05	07	09	12	18	25	34	44	66	84	95	99
7	2.115	05	07	10	14	21	30	41	53	76	91	98	*
8	2.089	06	07	10	16	24	35	47	60	83	95	99	
9	2.070	06	08	11	18	27	40	54	67	88	98	*	
10	2.055	06	08	12	20	31	45	60	73	92	99		
11	2.043	06	08	13	22	34	49	65	79	95	99		
12	2.033	06	09	14	24	38	54	70	83	97	*		
13	2.025	06	09	15	26	41	58	74	87	98			
14	2.018	06	09	17	29	45	62	78	90	99			
15	2.013	06	10	18	31	48	66	82	92	99			
16	2.008	06	10	19	33	51	70	85	94	*			
17	2.004	06	10	20	35	54	73	87	95				
18	2.000	06	11	21	37	57	76	90	97				
19	1.996	06	11	22	40	60	79	91	97				
20	1.993	06	12	23	42	63	82	93	98				
21	1.990	07	12	25	44	66	84	94	99				
22	1.988	07	13	26	46	68	86	95	99				
23	1.986	07	13	27	48	71	88	96	99				
24	1.984	07	13	28	50	73	89	97	99				
25	1.982	07	14	29	52	75	91	98	*				
26	1.980	07	14	31	54	77	92	98					
27	1.978	07	15	32	56	79	93	99					
28	1.977	07	15	33	58	81	94	99					
29	1.976	07	16	34	60	83	95	99					
30	1.974	07	16	36	62	84	96	99					
31	1.973	07	17	37	64	86	96	99					
32	1.972	07	17	38	65	87	97	*					
33	1.971	08	18	39	67	88	97						
34	1.970	08	18	41	69	89	98						
35	1.969	08	19	42	70	90	98						
36	1.968	08	19	43	72	91	98						
37	1.967	08	20	44	73	92	99						
38	1.967	08	20	46	75	93	99						
39	1.966	08	21	47	76	94	99						

Table 8.3.18 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.965	08	21	48	77	94	99	*	*	*	*	*	*
42	1.964	08	22	50	80	95	99						
44	1.963	08	23	53	82	96	*						
46	1.962	09	25	55	84	97							
48	1.961	09	26	57	86	98							
50	1.960	09	27	59	87	98							
52	1.959	09	28	61	89	99							
54	1.958	09	29	63	90	99							
56	1.957	09	30	65	91	99							
58	1.957	10	31	67	92	99							
60	1.956	10	32	69	93	99							
64	1.955	10	34	72	95	*							
68	1.954	11	37	75	96								
72	1.953	11	39	78	97								
76	1.952	12	41	81	98								
80	1.952	12	43	83	98								
84	1.951	12	45	85	99								
88	1.950	13	48	87	99								
92	1.950	13	50	89	99								
96	1.949	14	52	90	*								
100	1.949	14	54	92									
120	1.947	17	63	96									
140	1.946	19	71	98									
160	1.945	22	78	99									
180	1.944	24	83	*									
200	1.944	27	88										
250	1.943	34	95										
300	1.942	41	98										
350	1.941	48	99										
400	1.941	54	*										
450	1.941	60											
500	1.940	66											
600	1.940	75											
700	1.940	82											
800	1.940	88											
900	1.940	92											
1000	1.939	95											

* Power values below this point are greater than .995.

Table 8.3.19

Power of F test at a = .05, u = 10

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.854	05	05	06	07	08	09	10	12	16	23	30	39
3	2.258	05	06	07	09	11	13	17	21	32	46	62	76
4	2.133	05	06	07	10	13	17	23	30	47	65	81	92
5	2.053	05	06	08	11	16	22	30	40	61	80	92	98
6	2.008	05	07	09	13	19	28	38	50	73	90	97	*
7	1.978	05	07	10	15	23	33	45	59	82	95	99	
8	1.956	06	07	11	17	27	39	53	67	88	98	*	
9	1.940	06	08	12	20	31	44	60	74	93	99		
10	1.928	06	08	13	22	34	50	66	80	96	*		
11	1.913	06	09	14	24	38	55	71	84	97			
12	1.910	06	09	15	27	42	60	76	88	98			
13	1.903	06	09	17	29	46	65	81	91	99			
14	1.898	06	10	18	32	50	69	84	94	*			
15	1.893	06	10	19	34	53	73	87	95				
16	1.889	06	11	20	37	57	76	90	97				
17	1.885	06	11	22	39	60	79	92	98				
18	1.882	06	12	23	42	64	82	94	98				
19	1.879	06	12	24	44	67	85	95	99				
20	1.877	07	12	26	47	69	87	96	99				
21	1.874	07	13	27	49	72	89	97	99				
22	1.872	07	13	29	51	75	91	98	*				
23	1.870	07	14	30	54	77	92	98					
24	1.869	07	14	31	56	79	93	99					
25	1.867	07	15	33	58	81	94	99					
26	1.866	07	15	34	60	83	95	99					
27	1.864	07	16	36	62	85	96	99					
28	1.863	07	17	37	64	86	97	*					
29	1.862	07	17	38	66	88	97						
30	1.861	07	18	40	68	89	98						
31	1.860	08	18	41	70	90	98						
32	1.859	08	19	43	72	91	99						
33	1.858	08	19	44	73	92	99						
34	1.857	08	20	45	75	93	99						
35	1.856	08	21	47	76	94	99						
36	1.856	08	21	48	78	95	99						
37	1.855	08	22	49	79	95	99						
38	1.854	08	22	51	81	96	*						
39	1.854	08	23	52	82	96							

Table 8.3.19 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.853	08	23	53	83	97	*	*	*	*	*	*	*
42	1.852	09	25	56	85	98							
44	1.851	09	26	58	87	98							
46	1.850	09	27	61	89	99							
48	1.849	09	28	63	90	99							
50	1.848	09	30	65	92	99							
52	1.848	10	31	67	93	99							
54	1.847	10	32	69	94	*							
56	1.846	10	33	71	95								
58	1.846	10	35	73	96								
60	1.845	10	36	75	96								
64	1.845	11	38	78	97								
68	1.844	11	41	81	98								
72	1.843	12	43	84	99								
76	1.842	12	46	86	99								
80	1.842	13	48	88	99								
84	1.841	13	51	90	*								
88	1.841	14	53	92									
92	1.840	14	55	93									
96	1.840	15	57	94									
100	1.839	15	60	95									
120	1.838	18	69	98									
140	1.837	21	77	99									
160	1.836	24	84	*									
180	1.836	27	88										
200	1.835	30	92										
250	1.834	38	97										
300	1.834	46	99										
350	1.833	53	*										
400	1.833	60											
450	1.833	66											
500	1.832	72											
600	1.832	81											
700	1.832	88											
800	1.832	92											
900	1.832	95											
1000	1.832	97											

* Power values below this point are greater than .995.

Table 8.3.20

Power of F test at a = .05, u = 12

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.604	05	05	06	07	08	09	11	13	18	25	34	44
3	2.148	05	06	07	08	10	13	17	22	34	50	66	80
4	2.010	05	06	08	10	14	18	25	33	52	71	86	95
5	1.944	05	06	09	12	17	24	33	44	67	85	95	99
6	1.905	05	07	10	14	21	30	42	54	78	93	99	*
7	1.879	06	07	11	16	25	36	50	64	87	97	*	
8	1.860	06	08	12	19	29	43	58	72	92	99		
9	1.847	06	08	13	21	33	49	65	79	95	*		
10	1.836	06	08	14	24	38	55	71	85	98			
11	1.827	06	09	15	26	42	60	77	89	99			
12	1.821	06	09	17	29	46	65	81	92	99			
13	1.815	06	10	18	32	51	70	85	94	*			
14	1.810	06	10	19	35	55	74	88	96				
15	1.806	06	11	21	37	58	78	91	97				
16	1.802	06	11	22	40	62	81	93	98				
17	1.799	06	12	24	43	66	84	95	99				
18	1.796	07	12	25	46	69	87	96	99				
19	1.794	07	13	27	48	72	89	97	99				
20	1.792	07	13	28	51	75	91	98	*				
21	1.790	07	14	30	54	77	92	98					
22	1.788	07	14	31	56	80	94	99					
23	1.786	07	15	33	59	82	95	99					
24	1.785	07	15	34	61	84	96	99					
25	1.784	07	16	36	63	86	97	*					
26	1.782	07	17	37	65	88	97						
27	1.781	07	17	39	68	89	98						
28	1.780	07	18	41	70	90	98						
29	1.779	08	18	42	72	92	99						
30	1.778	08	19	44	73	93	99						
31	1.777	08	20	45	75	94	99						
32	1.776	08	20	47	77	94	99						
33	1.776	08	21	48	78	95	99						
34	1.775	08	22	50	80	96	*						
35	1.774	08	22	51	81	96							
36	1.774	08	23	53	83	97							
37	1.773	08	24	54	84	97							
38	1.773	08	24	55	85	98							
39	1.772	09	25	57	86	98							

Table 8.3.20 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.771	09	26	58	87	98	*	*	*	*	*	*	*
42	1.771	09	27	61	89	99							
44	1.770	09	28	63	91	99							
46	1.769	09	30	66	92	99							
48	1.768	10	31	68	94	*							
50	1.768	10	32	71	95								
52	1.767	10	34	73	96								
54	1.766	10	35	75	96								
56	1.766	11	36	77	97								
58	1.766	11	38	78	97								
60	1.765	11	39	80	98								
64	1.764	11	42	83	99								
68	1.763	12	45	86	99								
72	1.763	12	47	88	99								
76	1.762	13	50	90	*								
80	1.762	14	53	92									
84	1.761	14	55	93									
88	1.761	15	58	95									
92	1.760	15	60	96									
96	1.760	16	62	96									
100	1.760	16	65	97									
120	1.759	19	74	99									
140	1.758	23	82	*									
160	1.757	26	88										
180	1.756	29	92										
200	1.756	33	95										
250	1.755	41	98										
300	1.755	50	*										
350	1.754	58											
400	1.754	65											
450	1.754	71											
500	1.754	77											
600	1.753	86											
700	1.753	91											
800	1.753	95											
900	1.753	97											
1000	1.753	98											

* Power values below this point are greater than .995.

Table 8.3.21

Power of F test at a = .05, u = 15

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.352	05	05	06	07	08	10	12	14	20	28	39	51
3	1.992	05	06	07	09	11	15	19	25	39	57	74	87
4	1.880	05	06	08	11	15	20	28	37	58	78	92	98
5	1.826	05	07	09	13	19	27	38	50	74	91	98	*
6	1.794	05	07	10	15	23	34	47	61	85	96	*	
7	1.772	06	07	11	18	28	41	56	71	92	99		
8	1.757	06	08	12	21	33	48	65	79	96	*		
9	1.745	06	08	14	24	38	55	72	85	98			
10	1.736	06	09	15	27	43	61	78	90	99			
11	1.729	06	09	17	30	47	67	83	93	*			
12	1.724	06	10	18	33	52	72	87	96				
13	1.719	06	10	20	36	57	77	90	97				
14	1.715	06	11	21	39	61	81	93	98				
15	1.711	06	11	23	42	65	84	95	99				
16	1.708	06	12	25	45	69	87	96	99				
17	1.706	07	12	26	48	72	90	97	*				
18	1.704	07	13	28	51	76	92	98					
19	1.702	07	14	30	54	78	93	99					
20	1.700	07	14	31	57	81	95	99					
21	1.698	07	15	33	60	84	96	99					
22	1.696	07	16	35	63	86	97	*					
23	1.695	07	16	37	65	88	97						
24	1.694	07	17	39	68	89	98						
25	1.693	07	17	40	70	91	98						
26	1.692	07	18	42	72	92	99						
27	1.691	08	19	44	74	93	99						
28	1.690	08	20	46	76	94	99						
29	1.689	08	20	47	78	95	*						
30	1.688	08	21	49	80	96							
31	1.687	08	22	51	82	97							
32	1.687	08	22	52	83	97							
33	1.686	08	23	54	84	98							
34	1.686	08	24	56	86	98							
35	1.685	09	25	57	87	98							
36	1.684	09	25	59	88	99							
37	1.684	09	26	60	89	99							
38	1.683	09	27	62	90	99							
39	1.683	09	28	63	91	99							

Table 8.3.21 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.683	09	28	65	92	99	*	*	*	*	*	*	*
42	1.682	09	30	68	93	*							
44	1.681	10	32	70	95								
46	1.680	10	33	73	96								
48	1.680	10	35	75	97								
50	1.679	10	36	77	97								
52	1.679	11	38	79	98								
54	1.678	11	39	81	98								
56	1.678	11	41	83	99								
58	1.677	11	43	84	99								
60	1.677	12	44	86	99								
64	1.676	12	47	89	99								
68	1.676	13	50	91	*								
72	1.675	13	53	93									
76	1.675	14	56	94									
80	1.674	15	59	95									
84	1.674	15	62	96									
88	1.674	16	64	97									
92	1.673	17	67	98									
96	1.673	17	69	98									
100	1.673	18	71	99									
120	1.672	21	81	*									
140	1.671	25	88										
160	1.670	29	92										
180	1.670	33	96										
200	1.670	37	97										
250	1.669	47	99										
300	1.669	56	*										
350	1.668	64											
400	1.668	72											
450	1.668	78											
500	1.668	83											
600	1.667	91											
700	1.667	95											
800	1.667	97											
900	1.667	99											
1000	1.667	99											

* Power values below this point are greater than .995.

Table 8.3.22

Power of F test at a = .05, u = 24

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	1.964	05	06	06	08	09	11	14	17	26	39	53	69
3	1.737	05	06	07	10	13	18	24	32	52	73	88	96
4	1.663	05	07	09	12	18	26	37	49	74	91	98	*
5	1.627	05	07	10	15	24	35	49	64	88	98	*	
6	1.605	06	08	12	19	30	45	61	76	95	*		
7	1.590	06	08	13	22	36	54	71	85	98			
8	1.580	06	09	15	26	43	62	79	91	99			
9	1.572	06	09	17	30	49	69	86	95	*			
10	1.566	06	10	19	34	55	76	90	97				
11	1.561	06	11	21	38	61	81	94	99				
12	1.557	06	11	23	42	66	86	96	99				
13	1.554	06	12	25	47	71	89	98	*				
14	1.551	07	13	27	51	76	92	98					
15	1.549	07	13	29	54	80	94	99					
16	1.546	07	14	32	58	83	96	99					
17	1.545	07	15	34	62	86	97	*					
18	1.543	07	16	36	65	89	98						
19	1.542	07	16	38	69	91	99						
20	1.540	07	17	41	72	92	99						
21	1.539	07	18	43	75	94	99						
22	1.538	08	19	45	77	95	*						
23	1.537	08	20	48	80	96							
24	1.536	08	21	50	82	97							
25	1.536	08	22	52	84	98							
26	1.535	08	23	54	86	98							
27	1.534	08	24	57	87	99							
28	1.533	08	25	59	89	99							
29	1.533	09	25	61	90	99							
30	1.532	09	26	63	92	99							
31	1.532	09	27	65	93	*							
32	1.531	09	28	66	94								
33	1.531	09	29	68	94								
34	1.531	09	30	70	95								
35	1.530	09	31	72	96								
36	1.530	10	32	73	96								
37	1.529	10	34	75	97								
38	1.529	10	35	76	97								
39	1.529	10	36	78	98								

Table 8.3.22 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.529	10	37	79	98	*	*	*	*	*	*	*	*
42	1.528	11	39	82	99								
44	1.528	11	41	84	99								
46	1.527	11	43	86	99								
48	1.527	12	45	88	*								
50	1.526	12	47	90									
52	1.526	12	49	91									
54	1.526	13	51	92									
56	1.525	13	53	93									
58	1.525	13	55	94									
60	1.525	14	57	95									
64	1.524	14	60	97									
68	1.524	15	64	98									
72	1.523	16	67	98									
76	1.523	17	70	99									
80	1.523	18	73	99									
84	1.523	18	76	99									
88	1.522	19	79	*									
92	1.522	20	81										
96	1.522	21	83										
100	1.522	22	85										
120	1.521	26	92										
140	1.520	31	96										
160	1.520	36	98										
180	1.520	41	99										
200	1.519	47	*										
250	1.519	59											
300	1.519	70											
350	1.519	78											
400	1.518	85											
450	1.518	90											
500	1.518	94											
600	1.518	98											
700	1.518	99											
800	1.518	*											
900	1.518												
1000	1.518												

* Power values below this point are greater than .995.

Table 8.3.23

Power of F test at a = .10, u = 1

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	8.526	10	11	12	13	13	14	15	17	20	23	27	30
3	4.545	10	11	12	13	15	17	19	22	28	35	42	50
4	3.776	10	11	13	14	17	20	23	27	36	45	55	64
5	3.458	10	11	13	16	19	23	27	32	43	55	66	76
6	3.285	10	12	14	17	21	26	31	37	50	63	74	83
7	3.177	10	12	15	19	23	29	35	42	56	69	80	89
8	3.102	10	12	15	20	25	32	39	47	62	75	85	92
9	3.048	10	13	16	21	28	35	43	51	66	80	89	95
10	3.007	10	13	17	23	30	37	46	55	71	83	92	97
11	2.975	11	13	18	24	32	40	49	58	75	87	94	98
12	2.949	11	14	19	25	34	43	52	62	78	89	96	99
13	2.927	11	14	19	27	36	45	55	65	81	91	97	99
14	2.909	11	14	20	28	37	48	58	68	83	93	98	99
15	2.894	11	15	21	29	39	50	60	70	86	95	98	*
16	2.881	11	15	22	31	41	52	63	73	88	96	99	
17	2.869	11	15	23	32	43	54	65	75	89	97	99	
18	2.859	11	16	23	33	45	56	68	77	91	97	99	
19	2.850	11	16	24	34	46	58	70	79	92	98	*	
20	2.843	11	16	25	36	48	60	72	81	93	98		
21	2.836	11	17	26	37	50	62	73	83	94	99		
22	2.829	11	17	26	38	51	64	75	84	95	99		
23	2.823	11	18	27	39	53	66	77	86	96	99		
24	2.818	12	18	28	40	54	67	78	87	96	99		
25	2.813	12	18	29	42	56	69	80	88	97	99		
26	2.809	12	19	29	43	57	70	81	89	97	*		
27	2.805	12	19	30	44	58	72	83	90	98			
28	2.801	12	19	31	45	60	73	84	91	98			
29	2.797	12	20	32	46	61	74	85	92	98			
30	2.794	12	20	32	47	62	76	86	93	99			
31	2.791	12	20	33	48	63	77	87	93	99			
32	2.788	12	21	34	49	65	78	88	94	99			
33	2.786	12	21	34	50	66	79	89	95	99			
34	2.783	12	21	35	51	67	80	90	95	99			
35	2.781	13	22	36	52	68	81	90	96	99			
36	2.779	13	22	36	53	69	82	91	96	*			
37	2.777	13	22	37	54	70	83	92	96				
38	2.775	13	23	38	55	71	84	92	97				
39	2.773	13	23	38	56	72	85	93	97				

Table 8.3.23 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.771	13	24	39	57	73	85	93	97	*	*	*	*
42	2.768	13	24	40	59	75	87	94	98				
44	2.765	13	25	42	60	77	88	95	98				
46	2.762	14	26	43	62	78	90	96	99				
48	2.760	14	26	44	63	80	91	96	99				
50	2.758	14	27	45	65	81	92	97	99				
52	2.756	14	28	47	66	82	92	97	99				
54	2.754	14	28	48	68	84	93	98	99				
56	2.752	14	29	49	69	85	94	98	*				
58	2.750	15	30	50	71	86	95	98					
60	2.749	15	30	51	72	87	95	99					
64	2.746	15	31	53	74	89	96	99					
68	2.743	16	33	56	76	90	97	99					
72	2.741	16	34	58	78	92	98	99					
76	2.739	16	35	59	80	93	98	*					
80	2.738	17	36	61	82	94	99						
84	2.736	17	38	63	84	95	99						
88	2.735	17	39	65	85	96	99						
92	2.733	18	40	67	86	96	99						
96	2.732	18	41	68	88	97	99						
100	2.731	18	42	70	89	97	*						
120	2.727	20	48	76	93	99							
140	2.724	22	53	82	96	99							
160	2.721	24	57	86	98	*							
180	2.719	25	62	89	99								
200	2.718	27	65	92	99								
250	2.716	31	74	96	*								
300	2.714	35	80	98									
350	2.713	39	85	99									
400	2.712	42	89	*									
450	2.711	46	92										
500	2.711	49	94										
600	2.710	55	97										
700	2.709	61	98										
800	2.709	66	99										
900	2.708	70	*										
1000	2.708	74											

* Power values below this point are greater than .995.

Table 8.3.24

Power of F test at a = .10, u = 2

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	5.462	10	11	12	13	13	14	15	17	20	23	27	32
3	3.463	10	11	12	14	15	17	20	22	29	36	45	53
4	3.006	10	11	13	15	17	20	24	28	38	48	59	70
5	2.807	10	12	13	16	20	24	29	34	46	59	71	81
6	2.695	10	12	14	18	22	27	33	40	54	68	80	89
7	2.624	10	12	15	19	24	30	37	45	61	75	86	93
8	2.575	11	13	16	21	27	34	41	50	67	81	90	96
9	2.538	11	13	17	22	29	37	45	55	72	85	94	98
10	2.511	11	13	18	24	31	40	49	59	76	89	96	99
11	2.489	11	14	18	25	33	43	53	63	80	92	97	99
12	2.471	11	14	19	27	36	46	56	67	84	94	98	*
13	2.456	11	14	20	28	38	49	60	70	86	95	99	
14	2.444	11	15	21	30	40	51	63	73	89	97	99	
15	2.434	11	15	22	31	42	54	66	76	91	97	*	
16	2.425	11	16	23	32	44	56	68	79	92	98		
17	2.417	11	16	24	34	46	59	71	81	94	99		
18	2.410	11	16	24	35	48	61	73	83	95	99		
19	2.404	11	17	25	37	50	63	75	85	96	99		
20	2.398	12	17	26	38	52	65	77	87	97	*		
21	2.393	12	17	27	39	53	67	79	88	97			
22	2.389	12	18	28	41	55	69	81	90	98			
23	2.385	12	18	29	42	57	71	83	91	98			
24	2.381	12	19	29	43	59	73	84	92	99			
25	2.378	12	19	30	45	60	74	86	93	99			
26	2.375	12	19	31	46	62	76	87	94	99			
27	2.372	12	20	32	47	63	78	88	95	99			
28	2.369	12	20	33	48	65	79	89	95	99			
29	2.367	12	20	33	50	66	80	90	96	*			
30	2.365	12	21	34	51	68	82	91	96				
31	2.363	13	21	35	52	69	83	92	97				
32	2.361	13	22	36	53	70	84	93	97				
33	2.359	13	22	37	54	71	85	93	98				
34	2.357	13	22	37	55	73	86	94	98				
35	2.355	13	23	38	56	74	87	95	98				
36	2.354	13	23	39	57	75	88	95	98				
37	2.352	13	24	40	59	76	89	96	99				
38	2.351	13	24	40	60	77	89	96	99				
39	2.350	13	24	41	61	78	90	96	99				

Table 8.3.24 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.348	13	25	42	62	79	91	97	99	*	*	*	*
42	2.346	14	25	43	64	81	92	97	99				
44	2.344	14	26	45	65	82	93	98	*				
46	2.342	14	27	46	67	84	94	98					
48	2.341	14	28	48	69	85	95	99					
50	2.339	14	28	49	71	87	96	99					
52	2.338	15	29	50	72	88	96	99					
54	2.336	15	30	52	74	89	97	99					
56	2.335	15	31	53	75	90	97	99					
58	2.334	15	31	54	76	91	98	*					
60	2.333	15	32	55	78	92	98						
64	2.331	16	33	58	80	93	98						
68	2.329	16	35	60	82	95	99						
72	2.328	17	36	62	84	96	99						
76	2.326	17	38	65	86	96	99						
80	2.325	17	39	67	88	97	*						
84	2.324	18	40	69	89	98							
88	2.323	18	42	70	90	98							
92	2.322	18	43	72	92	99							
96	2.321	19	44	74	93	99							
100	2.321	19	45	75	93	99							
120	2.318	21	52	82	97	*							
140	2.315	23	57	87	98								
160	2.314	25	62	91	99								
180	2.313	27	67	94	*								
200	2.312	29	71	96									
250	2.310	33	80	98									
300	2.309	37	86	99									
350	2.308	42	90	*									
400	2.307	46	94										
450	2.307	49	96										
500	2.306	53	97										
600	2.306	60	99										
700	2.305	66	*										
800	2.305	71											
900	2.305	76											
1000	2.304	80											

* Power values below this point are greater than .995.

Table 8.3.25

Power of F test at a = .10, u = 3

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	4.191	10	11	12	12	13	15	16	17	20	25	29	35
3	2.924	10	11	12	14	15	18	20	23	31	39	49	59
4	2.606	10	11	13	15	18	21	25	30	41	53	65	76
5	2.462	10	12	14	17	20	25	30	37	50	64	77	87
6	2.381	10	12	15	18	23	29	35	43	59	73	85	93
7	2.327	11	12	15	20	26	32	40	49	66	81	91	96
8	2.291	11	13	16	22	28	36	45	54	72	86	94	98
9	2.264	11	13	17	23	31	40	49	59	78	90	97	99
10	2.243	11	14	18	25	33	43	54	64	82	93	98	*
11	2.226	11	14	19	27	36	46	58	68	86	95	99	
12	2.213	11	14	20	28	38	50	61	72	89	97	99	
13	2.202	11	15	21	30	41	53	65	76	91	98	*	
14	2.192	11	15	22	31	43	56	68	79	93	98		
15	2.184	11	16	23	33	45	59	71	82	95	99		
6	2.177	11	16	24	35	48	61	74	84	96	99		
7	2.171	11	16	25	36	50	64	77	86	97	*		
18	2.166	11	17	26	38	52	66	79	88	98			
19	2.162	12	17	27	39	54	69	81	90	98			
20	2.157	12	18	28	41	56	71	83	91	99			
21	2.154	12	18	29	43	58	73	85	93	99			
22	2.150	12	18	29	44	60	75	86	94	99			
23	2.147	12	19	30	46	62	77	88	95	99			
24	2.144	12	19	31	47	64	79	89	95	*			
25	2.142	12	20	32	48	66	80	90	96				
26	2.139	12	20	33	50	67	82	91	97				
27	2.137	12	21	34	51	69	83	91	97				
28	2.135	12	21	35	53	70	84	93	98				
29	2.133	13	21	36	54	72	86	94	98				
30	2.132	13	22	37	55	73	87	95	98				
31	2.130	13	22	38	57	75	88	95	99				
32	2.129	13	23	39	58	76	89	96	99				
33	2.127	13	23	39	59	77	90	96	99				
34	2.126	13	23	40	60	78	91	97	99				
35	2.124	13	24	41	61	79	91	97	99				
36	2.123	13	24	42	63	81	92	98	99				
37	2.122	13	25	43	64	82	93	98	*				
38	2.121	14	25	44	65	83	93	98					
39	2.120	14	26	45	66	84	94	98					

Table 8.3.25 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	2.119	14	26	45	67	84	94	99	*	*	*	*	*
42	2.118	14	27	47	69	86	95	99					
44	2.116	14	28	49	71	88	96	99					
46	2.115	14	28	50	73	89	97	99					
48	2.113	15	29	52	75	90	97	*					
50	2.112	15	30	53	76	91	98						
52	2.111	15	31	55	78	92	98						
54	2.110	15	32	56	79	93	99						
56	2.109	15	33	58	81	94	99						
58	2.108	16	33	59	82	95	99						
60	2.107	16	34	60	83	95	99						
64	2.106	16	36	63	85	96	99						
68	2.104	17	37	66	88	97	*						
72	2.103	17	39	68	89	98							
76	2.102	17	41	70	91	98							
80	2.101	18	42	72	92	99							
84	2.101	18	44	74	93	99							
88	2.100	19	45	76	94	99							
92	2.099	19	46	78	95	99							
96	2.098	20	48	80	96	*							
100	2.098	20	49	81	96								
120	2.096	22	56	87	99								
140	2.094	24	62	92	99								
160	2.093	26	68	95	*								
180	2.092	28	72	97									
200	2.091	30	77	98									
250	2.089	35	85	99									
300	2.088	40	91	*									
350	2.088	45	94										
400	2.087	50	97										
450	2.087	54	98										
500	2.087	58	99										
600	2.086	65	*										
700	2.086	71											
800	2.086	77											
900	2.085	81											
1000	2.085	85											

* Power values below this point are greater than .995.

Table 8.3.26

Power of F test at a = .10, u = 4

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	3.520	10	11	11	12	13	15	16	18	21	26	32	38
3	2.605	10	11	12	14	16	18	21	25	33	43	53	64
4	2.361	10	11	13	15	18	22	27	32	44	57	70	81
5	2.249	10	12	14	17	21	26	32	39	54	69	82	91
6	2.184	10	12	15	19	24	31	38	46	63	79	89	96
7	2.142	11	13	16	21	27	35	43	53	71	85	94	98
8	2.113	11	13	17	23	30	39	48	59	77	90	97	99
9	2.091	11	13	18	24	33	43	53	64	82	94	98	*
10	2.074	11	14	19	26	36	47	58	69	87	96	99	
11	2.061	11	14	20	28	38	50	62	73	90	97	*	
12	2.050	11	15	21	30	41	54	66	77	92	98		
13	2.041	11	15	22	32	44	57	70	81	94	99		
14	2.034	11	16	23	34	46	60	73	84	96	99		
15	2.027	11	16	24	35	49	63	76	86	97	*		
16	2.022	11	16	25	37	51	66	79	88	98			
17	2.017	11	17	26	39	54	69	81	90	98			
18	2.012	12	17	27	41	56	71	84	92	99			
19	2.009	12	18	28	42	58	74	86	93	99			
20	2.005	12	18	29	44	61	76	87	94	99			
21	2.002	12	19	30	46	63	78	89	95	*			
22	1.999	12	19	31	47	65	80	90	96				
23	1.997	12	20	32	49	67	82	92	97				
24	1.994	12	20	33	51	69	83	93	97				
25	1.992	12	21	34	52	70	85	94	98				
26	1.990	12	21	35	54	72	86	95	98				
27	1.989	13	21	36	55	74	87	95	99				
28	1.987	13	22	37	57	75	89	96	99				
29	1.986	13	22	38	58	77	90	97	99				
30	1.984	13	23	39	60	78	91	97	99				
31	1.983	13	23	40	61	79	92	97	99				
32	1.982	13	24	41	62	81	92	98	*				
33	1.980	13	24	42	64	82	93	98					
34	1.979	13	25	43	65	83	94	98					
35	1.978	13	25	44	66	84	94	99					
36	1.977	14	26	45	67	85	95	99					
37	1.977	14	26	46	69	86	96	99					
38	1.976	14	26	47	70	87	96	99					
39	1.975	14	27	48	71	88	96	99					

Table 8.3.26 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.974	14	27	49	72	89	97	99	*	*	*	*	*
42	1.973	14	28	51	74	90	97	*					
44	1.971	14	29	52	76	91	98						
46	1.970	15	30	54	78	93	98						
48	1.969	15	31	56	79	94	99						
50	1.968	15	32	57	81	94	99						
52	1.967	15	33	59	83	95	99						
54	1.966	16	34	61	84	96	99						
56	1.966	16	35	62	85	96	*						
58	1.965	16	36	64	86	97							
60	1.964	16	37	65	88	97							
64	1.963	17	38	68	90	98							
68	1.962	17	40	70	91	99							
72	1.961	18	42	73	93	99							
76	1.960	18	44	75	94	99							
80	1.959	19	45	77	95	*							
84	1.959	19	47	79	96								
88	1.958	19	48	81	97								
92	1.957	20	50	83	97								
96	1.957	20	52	84	98								
100	1.956	21	53	86	98								
120	1.954	23	60	91	99								
140	1.953	25	67	95	*								
160	1.952	28	73	97									
180	1.951	30	77	98									
200	1.951	32	82	99									
250	1.950	38	89	*									
300	1.949	43	94										
350	1.948	49	97										
400	1.948	53	98										
450	1.947	58	99										
500	1.947	62	*										
600	1.947	70											
700	1.947	76											
800	1.946	82											
900	1.946	86											
1000	1.946	89											

* Power values below this point are greater than .995.

Table 8.3.27

Power of F test at a = .10, u = 5

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	3.108	10	11	11	12	13	15	16	18	22	28	34	41
3	2.394	10	11	12	14	16	19	22	26	35	46	58	69
4	2.196	10	11	13	16	19	23	28	34	47	62	75	85
5	2.103	10	12	14	18	22	28	34	42	58	74	86	94
6	2.049	10	12	15	20	25	32	40	49	68	83	93	97
7	2.014	11	13	16	22	28	37	46	56	75	89	96	99
8	1.990	11	13	17	23	32	41	52	63	81	93	98	*
9	1.971	11	14	18	26	35	46	57	68	86	96	99	
10	1.957	11	14	19	28	38	50	62	73	90	97	*	
11	1.946	11	14	21	30	41	54	66	78	93	99		
12	1.937	11	15	22	32	44	57	70	81	95	99		
13	1.929	11	15	23	34	47	61	74	84	96	*		
14	1.923	11	16	24	36	50	64	77	87	97			
15	1.917	11	16	25	38	52	67	80	90	98			
16	1.912	11	17	26	40	55	70	83	92	99			
17	1.908	12	17	27	41	58	73	85	93	99			
18	1.905	12	18	29	43	60	76	87	95	99			
19	1.902	12	18	30	45	62	78	89	96	*			
20	1.899	12	19	31	47	65	80	91	96				
21	1.896	12	19	32	49	67	82	92	97				
22	1.894	12	20	33	51	69	84	93	98				
23	1.891	12	20	34	52	71	86	94	98				
24	1.890	12	21	35	54	73	87	95	99				
25	1.888	12	21	36	56	75	88	96	99				
26	1.886	13	22	38	57	76	90	97	99				
27	1.885	13	22	39	59	78	91	97	99				
28	1.883	13	23	40	61	79	92	98	99				
29	1.882	13	23	41	62	81	93	98	*				
30	1.881	13	24	42	64	82	94	98					
31	1.880	13	24	43	65	83	94	99					
32	1.879	13	25	44	66	85	95	99					
33	1.878	13	25	45	68	86	96	99					
34	1.877	14	26	46	69	87	96	99					
35	1.876	14	26	47	70	88	97	99					
36	1.875	14	27	48	72	89	97	99					
37	1.874	14	27	49	73	90	97	*					
38	1.874	14	28	50	74	90	98						
39	1.873	14	28	51	75	91	98						

Table 8.3.27 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.872	14	29	52	76	92	98	*	*	*	*	*	*
42	1.871	14	30	54	78	93	99						
44	1.870	15	31	56	80	94	99						
46	1.869	15	32	58	82	95	99						
48	1.868	15	33	60	83	96	99						
50	1.867	15	34	61	85	96	*						
52	1.866	16	35	63	86	97							
54	1.866	16	36	65	88	98							
56	1.865	16	37	66	89	98							
58	1.864	16	38	68	90	98							
60	1.864	17	39	69	91	99							
64	1.863	17	41	72	93	99							
68	1.862	18	43	75	94	99							
72	1.861	18	45	77	95	*							
76	1.860	19	46	79	96								
80	1.860	19	48	81	97								
84	1.859	20	50	83	98								
88	1.858	20	52	85	98								
92	1.858	21	54	86	98								
96	1.858	21	55	88	99								
100	1.857	22	57	89	99								
120	1.855	24	64	94	*								
140	1.854	27	71	97									
160	1.853	29	77	98									
180	1.853	32	81	99									
200	1.852	34	85	*									
250	1.851	40	92										
300	1.851	46	96										
350	1.850	52	98										
400	1.850	57	99										
450	1.849	62	*										
500	1.849	66											
600	1.849	74											
700	1.849	80											
800	1.849	84											
900	1.848	89											
1000	1.848	92											

* Power values below this point are greater than .995.

Table 8.3.28

Power of F test at a = .10, u = 6

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.827	10	11	11	12	13	14	17	19	23	29	36	44
3	2.243	10	11	12	14	16	19	23	27	37	49	61	73
4	2.075	10	11	13	16	20	24	30	36	50	66	79	89
5	1.996	10	12	14	18	23	29	36	45	62	78	89	96
6	1.950	11	12	15	20	26	34	43	53	71	86	95	99
7	1.919	11	13	17	22	30	39	49	60	79	92	98	*
8	1.898	11	13	18	24	33	44	55	66	85	95	99	
9	1.882	11	14	19	27	37	48	61	72	89	97	*	
10	1.870	11	14	20	29	40	53	66	74	93	99		
11	1.860	11	15	21	31	43	57	70	81	95	99		
12	1.852	11	15	23	33	46	61	74	85	97	*		
13	1.846	11	16	24	35	50	64	78	88	98			
14	1.840	11	16	25	38	53	68	81	90	98			
15	1.835	11	17	26	40	56	71	84	92	99			
16	1.831	12	17	27	42	58	74	86	94	99			
17	1.827	12	18	29	44	61	77	88	95	*			
18	1.824	12	18	30	46	64	79	90	96				
19	1.821	12	19	31	48	66	82	92	97				
20	1.819	12	19	32	50	68	84	93	98				
21	1.817	12	20	34	52	71	85	94	98				
22	1.815	12	20	35	54	73	87	95	99				
23	1.813	12	21	36	56	75	89	96	99				
24	1.811	13	21	37	57	77	90	97	99				
25	1.810	13	22	38	59	78	91	97	99				
26	1.808	13	23	40	61	80	92	98	*				
27	1.807	13	23	41	63	82	93	98					
28	1.806	13	24	42	64	83	94	99					
29	1.805	13	24	43	66	84	95	99					
30	1.803	13	25	44	67	86	96	99					
31	1.802	13	25	46	69	87	96	99					
32	1.802	14	26	47	70	88	97	99					
33	1.801	14	26	48	71	89	97	*					
34	1.800	14	27	49	73	90	97						
35	1.799	14	27	50	74	91	98						
36	1.798	14	28	51	75	91	98						
37	1.798	14	29	52	76	92	98						
38	1.797	14	29	53	78	93	99						
39	1.797	14	30	54	79	94	99						

Table 8.3.28 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.796	15	30	55	80	94	99	*	*	*	*	*	*
42	1.795	15	31	57	82	95	99						
44	1.794	15	32	59	84	96	99						
46	1.793	15	33	61	85	97	*						
48	1.792	16	35	63	87	97							
50	1.791	16	36	65	88	98							
52	1.791	16	37	67	89	98							
54	1.790	16	38	68	91	99							
56	1.790	17	39	70	92	99							
58	1.789	17	40	71	92	99							
60	1.789	17	41	73	93	99							
64	1.788	18	43	76	95	*							
68	1.787	18	45	78	96								
72	1.786	19	47	81	97								
76	1.785	19	49	83	98								
80	1.785	20	51	85	98								
84	1.784	20	53	86	99								
88	1.784	21	55	88	99								
92	1.783	21	57	89	99								
96	1.783	22	58	91	99								
100	1.783	22	60	92	*								
120	1.781	25	68	96									
140	1.780	28	75	98									
160	1.779	31	80	99									
180	1.779	33	85	*									
200	1.778	36	89										
250	1.778	43	94										
300	1.777	49	97										
350	1.777	55	99										
400	1.776	60	*										
450	1.776	66											
500	1.776	70											
600	1.776	78											
700	1.775	84											
800	1.775	89											
900	1.775	92											
1000	1.775	94											

* Power values below this point are greater than .995.

Table 8.3.29

Power of F test at a = .10, u = 8

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.469	10	11	11	12	14	15	17	20	25	33	41	50
3	2.038	10	11	13	15	17	21	25	30	41	55	68	80
4	1.909	10	12	14	17	21	26	32	40	56	72	85	93
5	1.847	10	12	15	19	25	32	40	49	68	84	94	98
6	1.811	11	13	16	21	29	37	48	58	78	91	98	*
7	1.787	11	13	17	24	33	43	55	66	85	95	99	
8	1.770	11	14	19	26	36	48	61	73	90	98	*	
9	1.757	11	14	20	29	40	53	67	79	94	99		
10	1.747	11	15	21	31	44	58	72	83	96	99		
11	1.740	11	15	23	34	48	63	76	87	98	*		
12	1.733	11	16	24	36	51	67	80	90	99			
13	1.728	11	16	26	39	55	71	84	93	99			
14	1.723	11	17	27	41	58	74	87	94	99			
15	1.720	12	18	28	44	61	77	89	96	*			
16	1.716	12	18	30	46	64	80	91	97				
17	1.713	12	19	31	48	67	83	93	98				
18	1.711	12	19	33	51	70	85	94	98				
19	1.709	12	20	34	53	72	87	95	99				
20	1.707	12	20	35	55	75	89	96	99				
21	1.705	12	21	37	57	77	91	97	99				
22	1.703	13	22	38	59	79	92	98	*				
23	1.702	13	22	40	61	81	93	98					
24	1.700	13	23	41	63	83	94	99					
25	1.699	13	24	42	65	84	95	99					
26	1.698	13	24	44	67	86	96	99					
27	1.697	13	25	45	69	87	96	99					
28	1.696	13	25	46	70	88	97	*					
29	1.695	13	26	48	72	90	97						
30	1.694	14	27	49	74	91	98						
31	1.693	14	27	50	75	92	98						
32	1.692	14	28	52	76	92	99						
33	1.692	14	29	53	78	93	99						
34	1.691	14	29	54	79	94	99						
35	1.691	14	30	55	80	95	99						
36	1.690	14	30	56	81	95	99						
37	1.689	15	31	58	83	96	99						
38	1.689	15	32	59	84	96	*						
39	1.688	15	32	60	85	97							

Table 8.3.29 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.688	15	33	61	86	97	*	*	*	*	*	*	*
42	1.687	15	34	63	87	98							
44	1.686	16	35	65	89	98							
46	1.686	16	37	67	90	99							
48	1.685	16	38	69	92	99							
50	1.684	16	39	71	93	99							
52	1.684	17	40	73	94	99							
54	1.683	17	42	75	94	99							
56	1.683	17	43	76	95	*							
58	1.682	18	44	78	96								
60	1.682	18	45	79	96								
64	1.681	18	48	82	97								
68	1.681	19	50	84	98								
72	1.680	20	52	86	99								
76	1.679	20	54	88	99								
80	1.679	21	56	90	99								
84	1.679	21	58	91	99								
88	1.678	22	60	93	*								
92	1.678	23	62	94									
96	1.677	23	64	95									
100	1.677	24	66	95									
120	1.676	27	74	98									
140	1.675	30	81	99									
160	1.675	33	86	*									
180	1.674	36	90										
200	1.674	39	93										
250	1.673	47	97										
300	1.673	54	99										
350	1.672	61	*										
400	1.672	66											
450	1.672	72											
500	1.672	76											
600	1.671	84											
700	1.671	89											
800	1.671	93											
900	1.671	96											
1000	1.671	97											

* Power values below this point are greater than .995.

Table 8.3.30

Power of F test at a = .10, u = 10

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.248	10	11	12	13	14	16	18	21	27	36	45	51
3	1.904	10	11	13	15	18	22	26	32	45	60	74	85
4	1.799	10	12	14	17	22	28	35	43	61	78	90	96
5	1.747	11	12	15	20	26	34	44	54	74	89	96	99
6	1.717	11	13	17	23	31	41	52	63	83	95	99	*
7	1.697	11	13	18	25	35	47	59	71	90	98	*	
8	1.683	11	14	20	28	39	53	66	78	94	99		
9	1.672	11	15	21	31	44	58	72	84	96	*		
10	1.664	11	15	23	34	48	63	77	88	98			
11	1.657	11	16	24	37	52	68	82	91	99			
12	1.652	11	16	26	39	56	72	85	93	99			
13	1.648	11	17	27	42	59	76	88	95	*			
14	1.644	12	18	29	45	63	79	91	97				
15	1.641	12	18	30	47	66	82	93	98				
16	1.638	12	19	32	50	69	85	94	98				
17	1.635	12	20	33	53	72	87	96	99				
18	1.633	12	20	35	55	75	89	97	99				
19	1.631	12	21	37	57	78	91	98	*				
20	1.630	12	22	38	60	80	93	98					
21	1.628	13	22	40	62	82	94	99					
22	1.627	13	23	41	64	84	95	99					
23	1.625	13	24	43	66	86	96	99					
24	1.624	13	24	44	68	87	97	99					
25	1.623	13	25	46	70	89	97	*					
26	1.622	13	26	47	72	90	98						
27	1.621	14	26	49	74	91	98						
28	1.620	14	27	50	76	92	98						
29	1.620	14	28	52	77	93	99						
30	1.619	14	28	53	79	94	99						
31	1.618	14	29	55	80	95	99						
32	1.618	14	30	56	81	95	99						
33	1.617	14	31	57	83	96	99						
34	1.616	15	31	59	84	96	*						
35	1.616	15	32	60	85	97							
36	1.615	15	33	61	86	97							
37	1.615	15	33	62	87	98							
38	1.615	15	34	64	88	98							
39	1.614	15	35	65	89	98							

Table 8.3.30 *(continued)*

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.614	15	35	66	90	98	*	*	*	*	*	*	*
42	1.613	16	37	68	91	99							
44	1.612	16	38	70	93	99							
46	1.612	16	40	72	94	99							
48	1.611	17	41	74	95	*							
50	1.611	17	42	76	95								
52	1.610	17	44	78	96								
54	1.610	18	45	80	97								
56	1.609	18	46	81	97								
58	1.609	18	48	83	98								
60	1.609	19	49	84	98								
64	1.608	19	52	86	99								
68	1.607	20	54	89	99								
72	1.607	21	56	90	99								
76	1.607	21	59	92	*								
80	1.606	22	61	93									
84	1.606	23	63	94									
88	1.605	23	66	95									
92	1.605	24	68	96									
96	1.605	25	70	97									
100	1.605	25	71	97									
120	1.604	29	79	99									
140	1.603	32	86	*									
160	1.603	36	90										
180	1.602	39	93										
200	1.602	43	96										
250	1.601	51	99										
300	1.601	59	*										
350	1.600	66											
400	1.600	72											
450	1.600	77											
500	1.600	81											
600	1.600	88											
700	1.600	93											
800	1.599	96											
900	1.599	98											
1000	1.599	99											

* Power values below this point are greater than .995.

Table 8.3.31

Power of F test at a = .10, u = 12

n	F_c	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	2.097	10	11	11	13	15	17	19	22	29	39	49	61
3	1.809	10	11	13	15	19	23	28	34	49	65	79	89
4	1.719	10	12	14	18	23	30	38	47	66	82	93	98
5	1.675	11	12	16	21	28	37	47	58	78	92	98	*
6	1.649	11	13	17	24	33	44	56	68	87	97	99	
7	1.631	11	14	19	27	37	50	64	76	93	99	*	
8	1.619	11	14	20	30	42	56	70	82	96	*		
9	1.610	11	15	22	33	47	62	76	87	98			
10	1.603	11	16	24	36	51	68	81	91	99			
11	1.597	11	16	25	39	56	72	86	94	99			
12	1.592	11	17	27	42	60	77	89	96	*			
13	1.588	12	18	29	45	64	80	92	97				
14	1.585	12	18	31	48	67	84	94	98				
15	1.582	12	19	32	51	71	86	95	99				
16	1.580	12	20	34	54	74	89	96	99				
17	1.578	12	20	36	56	77	91	97	*				
18	1.576	12	21	37	59	79	92	98					
19	1.574	13	22	39	62	82	94	99					
20	1.573	13	23	41	64	84	95	99					
21	1.571	13	23	43	66	86	96	99					
22	1.570	13	24	44	69	88	97	*					
23	1.569	13	25	46	71	89	97						
24	1.568	13	26	48	73	91	98						
25	1.567	13	26	49	75	92	98						
26	1.566	14	27	51	77	93	99						
27	1.565	14	28	52	78	94	99						
28	1.565	14	29	54	80	95	99						
29	1.564	14	29	55	81	95	99						
30	1.563	14	30	57	83	96	*						
31	1.563	14	31	58	84	97							
32	1.562	15	32	60	85	97							
33	1.562	15	32	61	87	98							
34	1.561	15	33	63	88	98							
35	1.561	15	34	64	89	98							
36	1.560	15	35	65	90	99							
37	1.560	15	35	67	90	99							
38	1.560	16	36	68	91	99							
39	1.559	16	37	69	92	99							

Table 8.3.31 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.559	16	38	70	93	99	*	*	*	*	*	*	*
42	1.558	16	39	73	94	99							
44	1.558	17	41	75	95	*							
46	1.557	17	42	77	96								
48	1.557	17	44	79	97								
50	1.556	18	45	81	97								
52	1.556	18	47	82	98								
54	1.555	18	48	84	98								
56	1.555	19	50	85	99								
58	1.555	19	51	86	99								
60	1.554	19	53	88	99								
64	1.554	20	55	90	99								
68	1.553	21	58	92	*								
72	1.553	22	60	93									
76	1.553	22	63	95									
80	1.552	23	65	96									
84	1.552	24	68	96									
88	1.552	24	70	97									
92	1.551	25	72	98									
96	1.551	26	74	98									
100	1.551	27	76	99									
120	1.550	31	84	*									
140	1.549	34	89										
160	1.549	38	93										
180	1.549	42	96										
200	1.548	46	97										
250	1.548	55	99										
300	1.548	63	*										
350	1.547	70											
400	1.547	76											
450	1.547	81											
500	1.547	85											
600	1.547	91											
700	1.547	95											
800	1.546	97											
900	1.546	99											
1000	1.546	99											

* Power values below this point are greater than .995.

Table 8.3.32

Power of F test at a = .10, u = 15

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	1.940	10	11	12	13	15	17	20	24	32	43	55	67
3	1.707	10	11	13	16	20	24	30	38	54	71	85	93
4	1.633	10	12	15	19	25	32	41	51	72	87	96	99
5	1.596	11	13	16	22	30	40	52	63	84	95	99	*
6	1.574	11	13	18	25	35	48	61	74	91	98	*	
7	1.560	11	14	20	29	41	55	69	81	96	99		
8	1.549	11	15	22	32	46	62	76	87	98	*		
9	1.541	11	15	23	36	51	68	82	92	99			
10	1.535	11	16	25	39	56	73	86	95	*			
11	1.531	12	17	27	43	61	78	90	97				
12	1.527	12	18	29	46	65	82	93	98				
13	1.523	12	18	31	49	69	85	95	99				
14	1.521	12	19	33	52	73	88	96	99				
15	1.518	12	20	35	56	76	91	97	*				
16	1.516	12	21	37	59	79	93	98					
17	1.514	12	22	39	62	82	94	99					
18	1.513	13	22	41	64	85	96	99					
19	1.511	13	23	43	67	87	97	99					
20	1.510	13	24	45	70	89	97	*					
21	1.509	13	25	46	72	90	98						
22	1.508	13	26	48	74	92	98						
23	1.507	13	26	50	76	93	99						
24	1.506	14	27	52	78	94	99						
25	1.505	14	28	54	80	95	99						
26	1.504	14	29	56	82	96	*						
27	1.504	14	30	57	83	97							
28	1.503	14	31	59	85	97							
29	1.503	15	32	61	86	98							
30	1.502	15	32	62	88	98							
31	1.502	15	33	64	89	98							
32	1.501	15	34	65	90	99							
33	1.501	15	35	67	91	99							
34	1.500	15	36	68	92	99							
35	1.500	16	37	70	93	99							
36	1.500	16	38	71	93	99							
37	1.499	16	39	72	94	*							
38	1.499	16	39	74	95								
39	1.499	16	40	75	95								

Table 8.3.32 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.498	17	41	76	96	*	*	*	*	*	*	*	*
42	1.498	17	43	78	97								
44	1.497	17	45	80	97								
46	1.497	18	46	82	98								
48	1.496	18	48	84	98								
50	1.496	18	49	86	99								
52	1.496	19	51	87	99								
54	1.495	19	53	88	99								
56	1.495	20	54	90	99								
58	1.495	20	56	91	*								
60	1.494	20	57	92									
64	1.494	21	60	94									
68	1.494	22	63	95									
72	1.493	23	66	96									
76	1.493	24	68	97									
80	1.493	24	71	98									
84	1.492	25	73	98									
88	1.492	26	75	99									
92	1.492	27	77	99									
96	1.492	28	79	99									
100	1.491	29	81	99									
120	1.491	33	88	*									
140	1.490	37	93										
160	1.490	42	96										
180	1.490	46	98										
200	1.489	50	99										
250	1.489	60	*										
300	1.489	68											
350	1.488	75											
400	1.488	81											
450	1.488	86											
500	1.488	90											
600	1.488	95											
700	1.488	97											
800	1.488	99											
900	1.488	99											
1000	1.488	*											

* Power values below this point are greater than .995.

Table 8.3.33

Power of F test at a = .10, u = 24

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	1.689	10	11	12	14	16	19	23	28	40	54	69	81
3	1.536	10	12	14	17	22	29	37	46	66	84	94	99
4	1.485	11	12	16	21	29	39	51	63	84	96	99	*
5	1.460	11	13	18	26	36	49	63	76	93	99	*	
6	1.445	11	14	20	30	43	58	73	85	98	*		
7	1.434	11	15	22	34	50	67	82	92	99			
8	1.427	11	16	25	39	56	74	88	95	*			
9	1.422	11	17	27	43	62	80	92	98				
10	1.417	12	18	29	48	68	85	95	99				
11	1.414	12	19	32	52	73	89	97	99				
12	1.411	12	20	35	56	78	92	98	*				
13	1.409	12	21	37	60	81	94	99					
14	1.407	12	22	40	64	85	96	99					
15	1.405	13	23	42	67	88	97	*					
16	1.404	13	24	45	71	90	98						
17	1.402	13	25	47	74	92	99						
18	1.401	13	26	50	77	94	99						
19	1.400	13	27	52	79	95	99						
20	1.399	14	28	54	82	96	*						
21	1.399	14	29	57	84	97							
22	1.398	14	30	59	86	98							
23	1.397	14	31	61	88	98							
24	1.397	15	32	63	89	99							
25	1.396	15	33	65	91	99							
26	1.395	15	35	67	92	99							
27	1.395	15	36	59	93	99							
28	1.395	15	37	71	94	*							
29	1.394	16	38	73	95								
30	1.394	16	39	74	95								
31	1.393	16	40	76	96								
32	1.393	16	41	77	97								
33	1.393	17	42	79	97								
34	1.393	17	43	80	98								
35	1.392	17	45	82	98								
36	1.392	17	46	83	98								
37	1.392	17	47	84	99								
38	1.392	18	48	85	99								
39	1.391	18	49	86	99								

Table 8.3.33 *(continued)*

		f											
n	F_c	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
40	1.391	18	50	87	99	*	*	*	*	*	*	*	*
42	1.391	19	52	89	99								
44	1.391	19	54	91	*								
46	1.390	20	56	92									
48	1.390	20	58	93									
50	1.389	21	60	94									
52	1.389	21	62	95									
54	1.389	22	64	96									
56	1.389	22	66	97									
58	1.389	23	67	97									
60	1.388	23	69	98									
64	1.388	24	72	98									
68	1.388	25	75	99									
72	1.388	26	78	99									
76	1.387	27	80	*									
80	1.387	29	83										
84	1.387	30	85										
88	1.387	31	87										
92	1.387	32	88										
96	1.386	33	90										
100	1.386	34	91										
120	1.386	40	96										
140	1.385	45	98										
160	1.385	51	99										
180	1.385	56	*										
200	1.385	61											
250	1.384	72											
300	1.384	80											
350	1.384	87											
400	1.384	91											
450	1.384	94											
500	1.384	97											
600	1.384	99											
700	1.384	*											
800	1.384												
900	1.384												
1000	1.384												

* Power values below this point are greater than .995.

The 33 tables in this section yield power values for the **F** test when, in addition to the significance criterion (**a**) and ES (**f**), the degrees of freedom for the numerator of the **F** ratio (**u**) and sample size (**n**) are specified. They are most directly used to appraise the power of **F** tests in a completed research *post hoc*, but can, of course, be similarly used for a research *plan*, the details of which (e.g., significance criterion, sample size) can be varied to study consequences to power.

The tables give values for **a**, **u**, **f**, and **n**:

1. Significance Criterion, **a**. Since **F** is naturally nondirectional (see above, Section 8.1), 11 tables (for varying **u**) are provided at each of the **a** levels, .01, .05, and .10.

2. Degrees of Freedom of the Numerator of the **F** *Ratio,* **u**. At each significance criterion, a table is provided for each of the following 11 values of **u**: 1 (1) 6 (2) 12, 15, 24. For cases 0, 1, and 2, all of which involve a comparison of $\mathbf{k} = \mathbf{u} + 1$ means, the number of means which can be compared using the tables is thus $\mathbf{k} = 2$ (1) 7 (2) 13, 16, and 25. For tests on interactions (Case 3), **u** is the interaction **df**, and equals $(\mathbf{k}-1)(\mathbf{r}-1)$, or $(\mathbf{k}-1)(\mathbf{r}-1)(\mathbf{p}-1)$, etc., where **k**, **r**, **p** are the number of levels of interacting main effects. Thus, $\mathbf{u} = 12$ for the interaction of a 4×5 or a 3×7 or a 2×13 factorial design or the three-way interaction of a $2 \times 4 \times 5$, a $2 \times 3 \times 7$, or a $3 \times 3 \times 4$ factorial design.

For missing values of **u** (7, 9, 11, etc.), linear interpolation between tables will yield quite adequate approximations.

3. Effect Size, **f**. Provision is made for 12 values of **f**: .05 (.05) .40 (.10) .80. For Cases 0 and 2, **f** is simply defined as the standard deviation of standardized means [formula (8.2.1)]. Its definition is generalized for unequal **n** (Case 1) and for interactions (Case 3), and the relevant formulas are given in the sections dealing with those cases. For all applications, conventional levels have been proposed (Section 8.2.3), as follows:

small: $\mathbf{f} = .10$,

medium: $\mathbf{f} = .25$,

large: $\mathbf{f} = .40$.

4. Sample Size, **n**. This is, for Cases 0 and 2, the **n** for *each* of the **k** sample means being compared. For the other cases, **n** is a function of the sizes of the samples or "cells" involved; see Sections 8.3.2, 8.3.4. The power tables provide for $\mathbf{n} = 2$ (1) 40 (2) 60 (4) 100 (20) 200 (50) 500 (100) 1000. Here, too, linear interpolation is quite adequate.

The values in the body of the tables are power times 100, i.e., the percent of tests carried out under the specified conditions which will result in rejection of the null hypothesis. They are rounded to the nearest unit and are generally accurate to within one unit as tabled.

8.3.1 CASE 0: **k** MEANS WITH EQUAL **n**. The simplest case is the one-way analysis of variance of **k** samples, *each* with the same number of observations, **n** (Case 0). The **F** test is based on $\mathbf{u} = \mathbf{k} - 1$ numerator **df**, and $\mathbf{k}(\mathbf{n} - 1)$ denominator **df**. The power tables were designed for Case 0 conditions, and this section describes and illustrates their use under these conditions. Later sections describe their application with unequal **n**'s (Case 1), in factorial and other designs (Case 2), and for tests of interactions (Case 3).

In Case 0, the investigator posits an alternate hypothesis or ES in terms of **f**, the standard deviation of standardized means, by one or more of the following procedures:

1. By hypothesizing the **k** varying population means expressed in the raw unit of measurement, finding the standard deviation of these means, and dividing this by the estimated within-population standard deviation. This is a literal application of formula (8.2.1). (See example 8.8 in Section 8.3.4.)
2. By hypothesizing the range of the **k** means (**d**) and their pattern, and using the formulas of Section 8.2.1. or the $\mathbf{c}_j$ values of Table 8.2.1 to convert **d** to **f**.
3. By hypothesizing the ES as a proportion of the total variance for which population membership accounts (η^2) or as a correlation ratio (η), and using the formulas of Section 8.2.2 [particularly formula (8.2.22)] or Table 8.2.2 to convert η or η^2 to **f**.
4. With experience, or perhaps by using the proposed operational definitions of small, medium, and large **f** values as a framework, he can work directly with **f**, i.e., simply directly specify his alternate hypothesis or ES by selecting an appropriate value of **f**.

Since the specification of a value of **f** which correctly reflects the investigator's ES expectations is crucial, cross-checking among the above routes is recommended. Thus, for example, having reached an **f** by specifying an η^2, it would be worthwhile to determine what range of means (**d**) for a given anticipated pattern that value of **f** implies, and to ascertain whether this **d** is consistent with expectation.

Once **f** is selected, the rest is simple in Case 0 applications. Find the table for the **a** and **u** ($= \mathbf{k} - 1$) of the problem and locate **n**, the common sample size, and **f**. This determines their power ($\times$ 100). For nontabulated **f** or **u**, linear interpolation is reasonably accurate.

Illustrative Examples

8.1 An educational psychologist performs an experiment in which $\mathbf{k} = 4$ different teaching methods are to be contrasted. A total of $\mathbf{N} = 80$ pupils are randomly assigned to samples of $\mathbf{n} = 20$ pupils per methods group and are tested on an achievement criterion test following instruction. The resulting data are tested by an overall **F** test of a one-way analysis of variance design, using an $\mathbf{a} = .05$ significance criterion.

In setting the ES which she expects in the population (i.e., the alternate hypothesis), she believes that the 4 means should span a range **d** of three-quarters of a within-population standard deviation. This judgment is based on past experience and knowledge of the characteristics of the teaching methods. On this basis, she further expects that the four means will be about equally spaced along this range, thus in Pattern 2 (Section 8.2.1). From Table 8.2.1, she reads that for $\mathbf{k} = 4$ in Pattern 2, $\mathbf{f} = .373\mathbf{d}$, so that, given an anticipated $\mathbf{d} = .75$, $\mathbf{f} = .373(.75) = .280$. Having reached this value, she cross-checks by noting [from formula (8.2.19)] that this implies an $\eta^2 = \mathbf{f}^2/(1 + \mathbf{f}^2) = .280^2/(1 + .280^2) = .0727$, i.e., about $7\frac{1}{4}\%$ of the measure's total variance is accounted for by group membership, or in correlation ratio terms, $\eta = \sqrt{.0727} = .270$. She observes further that $\mathbf{f} = .280$ is just slightly above the operational definition of a medium ES ($\mathbf{f} = .25$). She accepts the results of this cross-checking as consonant with her expectations. The necessary specifications for determining the power of the **F** test are complete. Note that in a one-way analysis of variance on **k** "levels," the numerator **df** are $\mathbf{u} = \mathbf{k} - 1 = 3$. Thus,

$$\mathbf{a} = .05, \quad \mathbf{u} = 3, \quad \mathbf{f} = .28, \quad \mathbf{n} = 20.$$

In Table 8.3.14 for $\mathbf{a} = .05$ and $\mathbf{u} = 3$, at row $\mathbf{n} = 20$, she finds power for column $\mathbf{f} = .25$ to be .43 and for $\mathbf{f} = .30$ to be .59. Linear interpolation yields (approximate) power of

$$.43 + \frac{(.28 - .25)}{(.30 - .25)}(.59 - .43) = .43 + .10 = .53.$$

Thus, if the standard deviation of the 4 standardized population means, **f**, is .28 of a within-population standard deviation, with $\mathbf{n} = 20$ cases per sample, the **F** test has had only a .53 probability of rejecting the null hypothesis at the .05 level. Note that the operative condition is the value of **f** of .28, whether the range and pattern of population means was as predicted or whether another range and pattern, which would yield the same **f**, applied.

An experiment whose power is as low as .53 for detecting its anticipated ES is relatively inconclusive when it fails to reject the null hypothesis. Given a population $\mathbf{f} = .28$, rather than $\mathbf{f} = 0$ as posited by the null hypothesis, it is a "toss-up" whether his results will be significant at the **a** and **n** conditions which obtain. Note that even if the **a** criterion were liberalized to .10, linear interpolation in Table 8.3.25 (for $\mathbf{a} = .10$, $\mathbf{u} = 3$) between $\mathbf{f} = .25$ and .30 gives approximate power at $\mathbf{n} = 20$ of only $.56 + .09 = .65$.

This problem has been presented as if the experiment were already completed (or at least committed), with a *post hoc* determination of power under the given conditions. See problem 8.9 below for a consideration of this problem as one of experimental *planning*, where, under stated conditions, the purpose is the determination of sample size to attain a specified power.

8.2 A large scale research on mental hospital treatment programs of chronic schizophrenics is undertaken by a psychiatric research team. A pool of $\mathbf{N} = 600$ suitable patients is randomly divided into 3 ($=\mathbf{k}$) equal samples, each assigned to a different building, and in each building a different microsocial system of roles, functions, responsibilities, and rewards of staff and patients is instituted following training. After a suitable interval, patients are assessed by the research team by means of behavior rating scales. The social-scientific "cost" of mistakenly rejecting the null hypothesis leads the team to decide on $\mathbf{a} = .01$. The team is split, however, on the question of how large an effect the difference in the three systems will have, some expecting that 5% of behavior rating variance will be accounted for by system membership, the others expecting 10%. Hence $\eta^2 = .05$ or .10. In their discussion, they agree in their expectation that the population means are at equal intervals, hence in Pattern 2 (but note that for $\mathbf{k} = 3$, Pattern 2 and Pattern 1 are the same). From Table 8.2.2, they note that at $\eta^2 = .05$, $\mathbf{f} = .229$, and at $\eta^2 = .10$, $\mathbf{f} = .333$. They determine, using the constants of Table 8.2.1, that the span of means for Pattern 2 for $\mathbf{f} = .229$ is $\mathbf{d}_2 = 2.45(.229) = .56$, and for $\mathbf{f} = .333$, $\mathbf{d}_2 = 2.45(.333) = .82$. Thus the proponents of $\eta^2 = .05$ expect a spread of the three means of a little more than half a within-population standard deviation, while the $\eta^2 = .10$ faction expect a spread of almost five-sixths of a σ. This translation brings them no closer to agreement. What is the power of the eventual **F** test under each of these two alternative hypotheses?

$$\mathbf{a} = .01, \qquad \mathbf{u} = \mathbf{k} - 1 = 2, \qquad \mathbf{f} = \begin{cases} .23 \\ .33 \end{cases}, \qquad \mathbf{n} = 200.$$

In Table 8.3.2 (for $\mathbf{a} = .01$, $\mathbf{u} = 2$) at row $\mathbf{n} = 200$, they find that at $\mathbf{f} = .20$, power is .98, and at $\mathbf{f} = .25$, power is greater than .995. This means they need have no dispute—if the $\mathbf{f} = .23$ ($\eta^2 = .05$) faction is right, power is about .99; if the $\mathbf{f} = .33$ ($\eta^2 = .10$) faction is right, power is greater than .995. If either is correct, they are virtually certain to reject the null hypothesis at $\mathbf{a} = .01$ with the **F** test.

In a circumstance like this, where there is "power to spare" (and assuming that the $\eta^2 = .05$ "pessimists" are not substantially overestimating the ES), there may be an opportunity to capitalize on these riches by enlarging on the experimental issues. For example, assume that there was a fourth microsocial system that had been a candidate for inclusion in the experiment and that adequate physical and staff resources are available for its inclusion. It might then be worth exploring the statistical power consequences of dividing the available 600 chronic patients into $\mathbf{k} = 4$ equal groups. Assuming no change in the conditions, and for the same **f** values, interpolation in Table 8.3.3 (for $\mathbf{a} = .01$, $\mathbf{k} - 1 = \mathbf{u} = 3$) shows that at $\mathbf{n} = 140$ (150 is not tabulated), power at $\mathbf{f} = .23$ is about .97 and at $\mathbf{f} = .33$, power again exceeds .995. Thus, this experiment could be enlarged at no substantial loss in power, assuming **f** is not materially lower than .23. But note that if **f** is really .15, the original $\mathbf{k} = 3$, $\mathbf{n} = 200$ experiment has still creditable power of .79 (Table 8.3.2), but the power of the revised $\mathbf{k} = 4$, $\mathbf{n} = 150$ experiment is only about .72 (interpolating between $\mathbf{n} = 140$ and 160 in Table 8.3.3).

8.3.2 CASE 1: **k** MEANS WITH UNEQUAL **n**. When the sample sizes ($\mathbf{n}_i$) drawn up from the **k** populations whose means ($\mathbf{m}_i$) are being compared are not all the same, no fundamental conceptual change occurs, but further attention to the definition of **f** is required and procedures for power analysis require accommodation from those of Case 0.

f was defined as the standard deviation of standardized means, σ_m/σ [formula (8.2.1)], where σ_m was given for equal **n** in formula (8.2.2) as

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{k} (\mathbf{m}_i - \mathbf{m})^2}{\mathbf{k}}}.$$

When **n**'s are not equal, it is no longer true that the reference point from which the "effects" are calculated, **m**, is a simple mean of the **k** population means, i.e., $\mathbf{m} = \sum \mathbf{m}_i/\mathbf{k}$, but rather a *weighted* mean of these means, the weight of each $\mathbf{m}_i$ being $\mathbf{p}_i$, the proportion of the total $\mathbf{N} = \sum \mathbf{n}_i$ which its sample $\mathbf{n}_i$ comprises. Thus, for Case 1

(8.3.1)
$$\mathbf{m} = \frac{\sum \mathbf{n}_i \mathbf{m}_i}{\mathbf{N}} = \sum \mathbf{p}_i \mathbf{m}_i.$$

The **m** for equal **n** is a special case of this formula, where all the $\mathbf{p}_i = \mathbf{n}/\mathbf{N} = \mathbf{n}/\mathbf{kn} = 1/\mathbf{k}$.

Similarly, in computing the standard deviations of the means, σ_m, the separate effects of the **k** populations, $\mathbf{m}_i - \mathbf{m}$, must be weighted by their proportionate sample sizes:

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{k} n_i(m_i - m)^2}{N}} = \sqrt{\sum_{i=1}^{k} p_i(m_i - m)^2} \,. \tag{8.3.2}$$

Here, too, the formula given for σ_m for equal **n** in the previous section (8.2.2) is a special case of formula (8.3.2), where all $p_i = 1/k$.

Thus, with the understanding that for unequal **n** each population mean "counts" to the extent of the relative proportion of its sample size, no change in the definition of **f** is required; it is the standard deviation of the (weighted) standardized means.

The implication of this weighting requires comment. If the populations whose means are extreme, i.e., have large $(m_i - m)^2$, also have large **n**'s relative to the others, **f** will be larger than with equal **n**; conversely, if extreme populations have small **n**'s, **f** will be smaller, This suggests that in circumstances where the researcher has reason to believe that certain of the **k** populations will provide particularly discrepant means, dividing the total **N** unequally with larger sample **n**'s drawn from these populations will increase **f** (over equal **n**), and thereby increase power.

This statistical fact, however, cannot necessarily be taken as a mandate to so design experiments. Its utilization depends on whether the purpose of the research is solely to (*a*) test with a view to reject the null hypothesis of equal population means, or whether it (*b*) seeks to reflect a "natural" population state of affairs. When there is no "natural" population, as when the populations are of different experimental manipulations of randomly assigned subjects, as in a true experiment, we are perforce in situation (*a*). When a natural population exists, our purpose may be either (*a*) or (*b*).

An illustration should clarify the distinction. In an experiment where the effect on a dependent variable of three different experimental conditions is under scrutiny, each condition is a systematic artificial creation of the experimenter. The populations are hypothetical collections of results of a given condition being applied to all subjects. Consider, by way of contrast, a survey research designed to inquire into differences among Protestants, Catholics, and Jews in scores on a scale of attitude toward the United Nations (AUN). Here there are also three populations, but population membership is not an artificial creation of the manipulative efforts of the investigator. These are natural populations, and their properties as *populations* include their relative sizes in their combined superpopulation. There is now a choice with regard to how sampling is to proceed. The investigator can draw a random sample of **N** cases of the total population and administer the AUN scale to all **N** cases, then sort them into religious groups. The proportions in each religious group will then not be equal, but reflect (within sampling error) the relative sizes of the religious affiliation populations. Alternatively, having decided to study a total of **N** cases, he can draw *equal* samples from each religion.

Now, assume that the Jews yield a small **p**, and that their AUN population mean is quite extreme. In the former sampling plan, the **f**, based on the small weight given the Jews, would be smaller than the **f** obtained with equal sample sizes, where the mean of the Jews would be weighted equally with the others. The larger **f** would have associated with it a larger η^2 (as well as greater power). But if η^2 is to be interpreted as giving the proportion of AUN variance associated with religion in the general population, i.e., *in the natural population,* where there are relatively few Jews, it is the first sampling plan and the smaller η^2 which is appropriate. The η^2 from equal sampling would have to be interpreted as the proportion of AUN variance associated with (artificially) equiprobable religious group membership. The equal-sampling η^2 is not objectionable if the investigator wishes to consider membership in a given religious group as an abstract effect quite apart from the relative frequency with which that effect (i.e., that religious group) occurs in the population, but it clearly cannot be referred to the natural population with its varying group frequencies.

On the other hand, assume that the purpose of the investigator is solely to determine *whether* religious population means differ on AUN, i.e., to determine the status of the overall null hypothesis. Thus, no issue as to the interpretation of η^2 need arise. On this assumption, if his alternate hypothesis gives him confidence that the population mean of the Jews will be discrepant, he may advantageously oversample Jews by having their **n** equal (or even draw a *larger* sample of Jews than of the other groups) in order to make **f** larger (if his alternate hypothesis is valid), and thus increase his power.

As has already been implied, the weighting of the population means does not change the meaning of η^2 nor disturb its relationship to **f**. Thus, formulas (8.2.16)–(8.2.22) and Table 8.2.2 all obtain for Case 1. This is *not* the case for the translation between **f** and **d** measures of range in the various patterns detailed in Section 8.2.1 [formulas (8.2.5)–(8.2.15) and Table 8.2.1]. The assumption throughout that material is one of equal sample sizes, and it is clear that any given **d** value for some pattern of **k** means will lead to differing **f**'s depending upon how the varying p_i are assigned to the m_i. The proposed conventions in regard to small, medium, and large **f** values continue to be applicable for Case 1 (except, of course, for their explication in terms of **d** values).

Finally, in Case 1, where there is no common **n** value to use in the power tables, one enters with their arithmetic mean:

$$\mathbf{n} = \frac{\sum_{i=1}^{k} \mathbf{n}_i}{\mathbf{k}} = \frac{\mathbf{N}}{\mathbf{k}}. \tag{8.3.3}$$

Aside from the use of the mean sample size, the procedure for the use of Table 8.3 is identical with that of Case 0.

Illustrative Examples

8.3 A university political science class has designed a poll to inquire into student opinion about the relative responsibilities and rights of local, state, and federal governments. An index score on centralism (CI) is derived and its relationship to various respondent characteristics is studied. One such characteristic is academic area, i.e., science, humanities, social science, etc., of which there are $\mathbf{k} = 6$ in all. Data are available on a random sample of 300 respondents drawn from the university student roster. In considering the ES that they anticipate, they note that since they intend to generalize to the natural population of the college and are sampling accordingly, they will have unequal sample sizes and their conception of **f** must take into account the differential weighting of effects in the σ_m of formula (8.3.2). So computed, they posit **f** at .15. They note ruefully that they expect the greatest effects [departures from the grand weighted mean of formula (8.3.1)] to come from the smallest academic area samples, and that if they had sampled the academic areas equally, they could anticipate an **f** of .20. However, sampling academic areas equally would result in inequalities on the "breaks" of the data which are to be studied, e.g., sex, political party affiliation, ethnic background. In any case, their interest lies in the correlates of CI in the "natural" university population.

What is the power at $\mathbf{a} = .05$ under the conditions which obtain, namely

$$\mathbf{a} = .05, \qquad \mathbf{u} = \mathbf{k} - 1 = 5, \qquad \mathbf{f} = .15, \qquad \mathbf{n} = \mathbf{N}/\mathbf{k} = 50.$$

Note that **n** is entered at the average sample size, $300/6 = 50$. Table 8.3.16 (for $\mathbf{a} = .05$, $\mathbf{u} = 5$) for row $\mathbf{n} = 50$, column $\mathbf{f} = .15$, indicates that power = .48. Clearly, the *a priori* probability of the **F** test's rejecting the null hypothesis given under these conditions is not very high.

Assume that it is undesirable to increase **a** to .10 (which would increase power to .61—see Table 8.3.27) or to draw a larger sample; is there some other possible strategem to improve the prognosis for this significance test? The following might be acceptable: The division of the cases into as many as six academic areas might be reconsidered, given the partially arbitrary nature of such a partitioning. The class might discover that a somewhat less fine discrimination into three more broadly defined academic areas such as science, humanities–arts, and engineering might be acceptable. Assume that under these conditions **f** [still based on the σ_m of formula (8.3.1)] is again computed to be about .15. The revised plan has the conditions

$$\mathbf{a} = .05, \qquad \mathbf{u} = 3 - 1 = 2, \qquad \mathbf{f} = .15, \qquad \mathbf{n} = 300/3 = 100.$$

In Table 8.3.13 for $\mathbf{a} = .05$ and $\mathbf{u} = 2$, $\mathbf{n} = 100$, and $\mathbf{f} = .15$, power = .64, a distinct improvement over the .48 value of the previous plan. If this process can, without doing violence to the issue, be carried a step further to a partitioning into two areas, and *if* the same **f** can be assumed, Table 8.3.12 (for $\mathbf{a} = .05$, $\mathbf{u} = 1$) gives power at $\mathbf{n} = 300/2 = 150$ for $\mathbf{f} = .15$ of about .74 (by linear interpolation). It must again be stressed that all this reasoning takes place without recourse to the data which are to be analyzed, i.e., we are in the area of planning the data analysis.

Thus, when there is some freedom available in the partitioning of a sample into groups, power considerations may advantageously enter into the decision. With **f** (and total **N**) constant, fewer groups and hence smaller **u** with larger **n** will result in increased power. Although **f** will not in general remain constant over changes in partitioning, this too may become a useful lever in planning analyses, since some partitions of the total sample will lead to larger anticipated **f** values, and hence greater power, than others. Therefore, when alternative partitions are possible, the investigator should seek the one whose combined effect on **u** and expected **f** is such as to maximize power. See problems 8.13 and 8.14 for further discussion.

8.4 As part of an inquiry into the differential effectiveness of psychiatric hospitals in a national system, an analysis is to be performed on the issue as to whether the psychiatric nurses in the various hospitals differ from hospital to hospital with regard to scores on an attitude scale of Social Restrictiveness (Cohen & Struening, 1963; 1964). There are $\mathbf{k} = 12$ psychiatric hospitals of wide geographic distribution which have supplied quasi-random samples of their nursing personnel of varying sizes, depending upon administrative considerations and the size of their nursing staffs. The total $\mathbf{N} = 326$, so that the average **n** per hospital is $326/12 = 27.2$. The investigators anticipate that the ES of hospital on attitude is of medium size, i.e., that $\mathbf{f} = .25$. They note that the **f** in question includes the differential weighting of the σ_m of formula (8.2.3), but since they have no reason to expect any relationship between the size of a hospital mean's discrepancy from the grand mean (i.e., the hospital's "effect") and the size of its sample, there is no need to modify the conception of a medium ES being operationalized by $\mathbf{f} = .25$.

What is the power of the **F** test on means at $\mathbf{a} = .05$? The conditions of the test, in summary, are

$$\mathbf{a} = .05, \quad \mathbf{u} = \mathbf{k} - 1 = 11, \quad \mathbf{f} = .25, \quad \mathbf{n} = 27.$$

There are no tables for $\mathbf{u} = 11$, so that interpolation between Tables 8.3.19 (for $\mathbf{a} = .05$, $\mathbf{u} = 10$) and 8.3.20 (for $\mathbf{a} = .05$, $\mathbf{u} = 12$) is necessary. Table 8.3.19 for row $\mathbf{n} = 27$ and column $\mathbf{f} = .25$ yields power of .85. Table 8.3.20 for the same **n** and **f** gives power of .89. Linear interpolation between these values yields a power estimate of .87. Thus, given that the (weighted) standard deviation of the standardized means of the populations of nurses in these 12 hospitals is .25, the probability that **F** will meet the $\mathbf{a} = .05$ criterion is .87, a value that would probably be deemed quite satisfactory.

8.3.3 CASE 2: FIXED MAIN EFFECTS IN FACTORIAL AND COMPLEX DESIGNS. In any experimental design of whatever structural complexity, a "fixed main effect" can be subjected to approximate power analysis with the aid of the tables of this chapter. In factorial, randomized blocks, split-plot, Latin square (etc.) designs, the **F** test on a fixed main effect involving **k** levels is a test of the equality of the **k** population means, whatever other fixed or random main or interaction effects may be included in the design (Winer, 1971; Hays, 1973; Edwards, 1972). We will illustrate the principles involved in this extension by examining power analysis of a main effect in a fixed factorial design. Except for a minor complication due to denominator **df**, and some qualification in the interpretation of η^2, this test proceeds as in Cases 0 and 1 above.

Consider, for example, an $\mathbf{I} \times \mathbf{J}$ factorial design, where there are $\mathbf{i} = 3$ levels of **I**, $\mathbf{j} = 4$ levels of **J**, and each of the $\mathbf{ij} = 12$ cells contains $\mathbf{n_c} = 10$ observations. The structure of the analysis in the usual model which includes interaction is:

Effect	df
I	$\mathbf{u_I} = \mathbf{i} - 1 = 2$
J	$\mathbf{u_J} = \mathbf{j} - 1 = 3$
I × J	$\mathbf{u_{I \times J}} = (\mathbf{i} - 1)(\mathbf{j} - 1) = 6$
Within cell (error)	$\mathbf{ij}(\mathbf{n_c} - 1) = 12(9) = 108$
Total	$\mathbf{ijn_c} - 1 = 119$

Now, consider the null hypothesis for the **J** effect, i.e., that the 4 population means of $\mathbf{J_1}$ through $\mathbf{J_4}$ are equal. The 4 sample means for **J** are each computed on $\mathbf{n_j} = \mathbf{in_c} = 3(10) = 30$ observations. (Similarly, each of the 3 means for **I** is computed on $\mathbf{n_I} = \mathbf{jn_c} = 4(10) = 40$ observations.) The minor complication arises at the point where one wants to determine the power of the test on **J** by applying the appropriate $\mathbf{u_J} = 3$ table at $\mathbf{n} = \mathbf{n_J} = 30$. This procedure is equivalent to ignoring the fact that the **I** main effect and $\mathbf{I} \times \mathbf{J}$ interaction exist in the design, i.e., a Case 0 test of 4 means, each of $\mathbf{n} = 30$. But the latter test has for its **F**-ratio *denominator* (within cell, or error) **df**, $4(30 - 1) = 116$. More generally, the denominator **df** presumed in the calculation of the table entries is, for **k** means each of **n** cases, $\mathbf{k}(\mathbf{n} - 1) = (\mathbf{u} + 1)(\mathbf{n} - 1)$. Thus, in this case, the table's value is based on 3 and 116 **df**, while the **F** test to be performed is for 3 and 108**df**.

To cope with this problem of the discrepancy in denominator (error) **df** between the presumption of a single source of nonerror variance of one-way design on which the tables are based and the varying numbers of sources of nonerror variance (main effects, interactions) of factorial and other complex designs, for all tests of effects in the latter, we adjust the **n** used for table entry to

$$\mathbf{n}' = \frac{\text{denominator } \mathbf{df}}{\mathbf{u} + 1} + 1. \tag{8.3.4}$$

The denominator **df** for a factorial design is the total **N** minus the total number of cells, and **u** is the **df** of the effect in question, as exemplified above for the $\mathbf{I} \times \mathbf{J}$ factorial design. Concretely, the **J** effect is tested as if it were based on samples of size

$$\mathbf{n}' = \frac{108}{3 + 1} + 1 = 28,$$

which together with the **f** value posited for the **J** effect, is used for entry into the appropriate table (for **a** and **u**) to determine power.

What happens to the interpretation of **f** when the basis of classification **K** into **k** levels is present together with others, as it is in factorial design? However complicated the factorial design, i.e., no matter how many other factors (**I**, **J**, etc.) and interactions ($\mathbf{K} \times \mathbf{I}$, $\mathbf{K} \times \mathbf{J}$, $\mathbf{K} \times \mathbf{I} \times \mathbf{J}$, etc.) may be involved, the definition of **f** for the **k** means of **K** remains the same—the standard deviation of the **k** standardized means, where the standardization is by the common within (cell) population standard deviation [formulas (8.2.1) and (8.2.2)]. Thus, there is no need to adjust one's conception of **f** for a set of **k** means when one moves from the one-way analysis of variance (Cases 0 and 1) to the case where additional bases of partitioning of the data exist. Furthermore, the translation between **f** and the **d** measures considered in 7.2.1 is also not affected. It is, however, necessary to consider the interpretation of η^2 in Case 2.

In Section 8.2.2, η^2 was defined as the proportion of the total variance made up by the variance of the means [formula 8.2.18)]. The total variance, in turn, was simply the sum of the within-population variance and the variance of the means [formula (8.2.17)]. The framework of that exposition was the analysis of variance into two components, between-populations and within-populations. In factorial design, the total variance is made up not only of the within (cell) population variance and the variance of the means of the levels of the factor under study, but also the variances of the means of the other factor(s) and also of the interactions. Therefore, the variance base of η^2 of formula (8.2.18), namely $\sigma^2 + \sigma_m{}^2$, is no longer the total variance, and the formulas involving η and η^2 [(8.2.19), (8.2.20), (8.2.22)] and Table 8.2.2 require the reinterpretation of η as a *partial* correlation ratio, and η^2 as a proportion, not of the total variance, but of the total from which there has been excluded (partialled out) the variance due to the other factor(s) and interactions.

This can be made concrete by reference to the $\mathbf{I} \times \mathbf{J}$ (3 × 4) factorial illustration. Consider the four population means of the levels of $\mathbf{J}$ and assume their $\mathbf{f_J}$ is .30. Assume further that $\mathbf{f_I}$ is .50 and $\mathbf{f_{I\times J}}$ is .20. When η^2 for $\mathbf{J}$ is computed from formula (8.2.19) (or looked up in Table 8.2.2):

$$\eta^2 = \frac{\mathbf{f}^2}{1+\mathbf{f}^2} = \frac{.30^2}{1+.30^2} = .0826,$$

the results for $\mathbf{J}$ clearly are not in the slightest affected by the size of the $\mathbf{I}$ or $\mathbf{I} \times \mathbf{J}$ effects. The η^2 for $\mathbf{J}$ in this design might be written in the conventional notation of partial correlation, with $\mathbf{Y}$ as the dependent variable under study, as $\eta^2{}_{\mathbf{YJ\cdot I,I\times J}}$, i.e., the proportion of the $\mathbf{Y}$ variance associated with $\mathbf{J}$ population membership, when variance due to $\mathbf{I}$ and to $\mathbf{I} \times \mathbf{J}$ is excluded from consideration. Thus, given $\mathbf{f_J} = .30$, the variance of the $\mathbf{J}$ means accounts for .0826 of the quantity made up of itself plus the within-cell population variance.

In higher order factorial designs, the η^2 computed from an $\mathbf{f}$ for a given source $\mathbf{J}$ might be represented as $\eta^2{}_{\mathbf{YJ}\cdot\text{all other}}$, the "all other" meaning all the other sources of total variance, main effects, and interactions. Each source's "size" may be assessed by such a partial PV. Because of their construction, however, they do not cumulate to a meaningful total.

The proposed operational definitions of small, medium, and large ES in terms of $\mathbf{f}$ have their usual meaning. When assessing power in testing the effects of the above $\mathbf{I} \times \mathbf{J}$ factorial, $\mathbf{f_I}$ and $\mathbf{f_J}$ (and also $\mathbf{f_{I\times J}}$—see Section 8.3.4) can each be set quite independently of the others (because of their partial nature), by using the operational definitions or by whatever other

means suit the investigator. They can, for example, be set by stating the alternative-hypothetical *cell* means and σ, and computing the resulting $\mathbf{f}$ values for all effects (illustrated in example 8.9 of the next section).

The scope of the present treatment precludes a detailed discussion of the power analysis of fixed effects in complex designs other than the factorial. Such analyses can be accomplished using the tables of this chapter if the following principles are kept in mind:

1. The basic ES index, $\mathbf{f}$, represents the standard deviation of *standardized* means, the standardization being accomplished by division by the appropriate σ. We have seen that for fixed factorial designs, σ is the square root of the within *cell* population variance. In other designs, and more generally, σ is the square root of the variance being estimated by the denominator ("error") mean square of the $\mathbf{F}$ test which is to be performed. For example, in repeated measurements designs using multiple groups of subjects ("split plot" designs), there are at least two error terms, (a) a "subjects within groups" or between-subjects error, and (b) an interaction term involving subjects, or within-subject error. In the definition of $\mathbf{f}$ for any source (i.e., set of means), the standardization or scaling of the σ_m will come from either (a) or (b), depending on whether the source is a between or a within source, just as will their $\mathbf{F}$ ratio denominators (Winer, 1971).

2. The adjustment to $\mathbf{n}'$ of formula (8.3.4) calls for the denominator $\mathbf{df}$, i.e., the $\mathbf{df}$ for the actual error term of the $\mathbf{F}$ ratio that is appropriate for the test of that source of variance in that design. For example, consider the test of the treatment effect in an unreplicated 6 × 6 Latin square (Edwards, 1972, pp. 285–317). Six treatment means, each based on $\mathbf{n} = 6$ observations, are to be compared, so $\mathbf{u} = 5$. Since the Latin square residual (error) mean square, which is the denominator of the $\mathbf{F}$ ratio, is based on $(\mathbf{n} - 1)(\mathbf{n} - 2) = 20$ $\mathbf{df}$, the $\mathbf{n}'$ for table entry is, from (8.3.4), $20/(6+1) + 1 = 3.86$. Power would then be found by linear interpolation between $\mathbf{n} = 3$ and 4 at the $\mathbf{f}$ value posited in the power table for $\mathbf{u} = 5$ for the specified $\mathbf{a}$ level.

Illustrative Examples

8.5 An experimental psychologist has designed an experiment to investigate the effect of genetic strain ($\mathbf{I}$) at $\mathbf{i} = 3$ levels and conditions of irradiation ($\mathbf{J}$) at $\mathbf{j} = 4$ levels on maze learning in rats. He draws 24 animals randomly from a supply of each genetic strain and apportions each strain sample randomly and equally to the four conditions, so that his 3 × 4 = 12 cells each contain a maze score for each of $\mathbf{n_c} = 6$ animals for a total $\mathbf{N}$ of 12(6) = 72 animals. The denominator $\mathbf{df}$ for the $\mathbf{F}$ tests in this analysis is therefore 72 − 12 = 60. He expects a medium ES for $\mathbf{I}$ and a large

ES for **J**, and following the operational definitions of Section 8.2.3, sets $\mathbf{f_I} = .25$ and $\mathbf{f_J} = .40$. Note that these values are standardized by the within cell population and each of the main effects is independent of the other. (The question of the **I** × **J** interaction is considered in the next section under Case 3.) What is the power of these two main effect **F** tests at the **a** = .05 criterion?

For the test on the equality of the mean maze scores for the 3 strains (**I**), **u** = **i** = 2, and each mean is taken over 24 animals. However, for table entry, we require the **n**′ of formula (8.3.4): 60/(2 + 1) + 1 = 21. Thus, the specifications are:

a = .05, **u** = 2, **f** = .25, **n**′ = 21.

Table 8.3.13 (**a** = .05, **u** = 2) at row **n** = 21 and column **f** = .25 indicates power of .40. The chances of detecting a medium effect in strain differences for these specifications are only two in five.

For a test of equality of means of the four irradiation conditions (**J**), **u** = **j** − 1 = 3, and each mean is taken over 18 animals. Again it is **n**′ of formula (8.3.4) that is required, and it is 60/(3 + 1) + 1 = 16. The specification summary for the test on **J** is thus:

a = .05, **u** = 3, **f** = .40, **n**′ = 16.

In Table 8.3.14 (**a** = 05, **u** = 3), at row **n** = 16 and column **f** = .40, he finds power = .75. The power of the test on irradiation conditions (**J**), given the large effect anticipated, is distinctly better than that for genetic strains (**I**); a probability of .75 of rejecting the null hypothesis means .75/.25, or three to one odds for rejection under these specifications.

8.6 An experiment in developmental social psychology is designed to study the effect of sex of experimenter (**S** at **s** = 2 levels), age of subject (**A** at **a** = 3 levels), instruction conditions (**C**, at **c** = 4), and their interactions (which are considered in the next section) on the persuasibility of elementary school boys. A total **N** of 120 subjects is assigned randomly (within age groups and equally) to the 2 × 3 × 4 = 24 cells of the design; thus, there are 5 cases in each cell. Expectations from theory and previous research lead the experimenter to posit, for each effect, the following ES for the three effects: $\mathbf{f_S} = .10$, $\mathbf{f_A} = .25$, and $\mathbf{f_C} = .40$. (Note that these **f** values imply *partial* η^2, respectively, of .01, .06, and .14.) Using as a significance criterion **a** = .05, what is the power of each of the main effects **F** tests?

This is a 2 × 3 × 4 fixed factorial design, and although we will not here consider the power testing of the four interaction effects (**S** × **A**, **S** × **C**, **A** × **C**, and **S** × **A** × **C**), they are part of the model (see Illustrative Example 8.7 in Section 8.3.4). The correct **df** for the denominator (within cell mean square) of all the **F** tests is 120 − 24 = 96.

For the test of the **S** effect, **u** = 2 − 1 = 1, and although each mean is based on 60 cases, the **n**′ for table entry is 96/(1 + 1) + 1 = 49. Thus, the specifications are

a = .05, **u** = 1, **f** = .10, **n**′ = 49.

In Table 8.3.12 for **a** = .05 and **u** = 1, at column **f** = .10, for both rows **n** = 48 and 50, power is given as .16. The probability of detecting **f** = .10 (a conventionally small effect) is very poor.

For the three age groups (hence **u** = 2), the **n**′ obtained by formula (8.3.4) is 96/(2 + 1) + 1 = 33. The specifications for the determination of the power of the **F** test on the **A** main effect are thus:

a = .05, **u** = 2, **f** = .25, **n**′ = 33.

In Table 8.3.13 (**a** = .05, **u** = 2), at row **n** = 33 and column **f** = .25, power = .59. Note that **f** = .25 is our conventional definition of a medium effect.

Finally, the test of the means of the four instruction conditions (hence **u** = 3) has for its **n**′ 96/(3 + 1) + 1 = 25. The specification summary:

a = .05, **u** = 3, **f** = .40, **n**′ = 25.

Table 8.3.14 at row **n** = 25, column **f** = .40 yields power of .93. Under these conditions, the **b** (Type II) error (1 − power) is about the same as the **a** (Type I) error, but note that a large effect has been posited.

In summary, the experimenter has a very poor (.16) expectation of detecting the small **S** effect, a no better than fair (.59) chance of detecting the medium **A** effect, and an excellent (.93) chance of finding a significant **C** effect, assuming the validity of his alternate hypotheses (i.e., his **f** values), **a** = .05, and **N** = 120. As an exercise, the reader may determine that changing the specifications to 6 cases per cell (**N** = 144), and leaving the other specifications unchanged, the tabled power values become .19 for **S**, .70 for **A**, and .97 for **C**. Note the inconsequential improvement this 20% increase in the size of the experiment has for the **S** and **C** effects, although bringing **A** from power of .59 to .70 might be worthwhile. Reaching significant power for **S** seems hopeless, but we have repeatedly seen that very large samples are required to obtain good power to detect small effects.

8.3.4 CASE 3: TESTS OF INTERACTIONS. A detailed exposition of interaction effects in experimental design is beyond the scope of this handbook; the reader is referred to one of the standard treatments (e.g., Hays, 1981; Winer, 1971; Edwards, 1972). We assume throughout equal $\mathbf{n_c}$ in the cells of the factorial.

For our present purposes, we note that an $\mathbf{R} \times \mathbf{C}$ interaction can be understood in the following ways:

1. Differences in effects between two levels of **R**, say $\mathbf{R_i}$ and $\mathbf{R_k}$ ($\mathbf{i}, \mathbf{k} =$ 1, 2, 3, . . . , $\mathbf{r}$; $\mathbf{i} < \mathbf{k}$) with regard to differences in pairs of **C**, say $\mathbf{C_j} - \mathbf{C_p}$ ($\mathbf{j}, \mathbf{p} = 1, 2, 3, \ldots, \mathbf{c}$; $\mathbf{j} < \mathbf{p}$). More simply, a contribution to an $\mathbf{R} \times \mathbf{C}$ interaction would be a difference between two levels of **R** with regard to a difference between two levels of **C**. Thus, if in the population, the sex difference (males minus females) in conditioning to sound ($\mathbf{C_j}$) is algebraically larger than the sex difference in conditioning to electric shock ($\mathbf{C_p}$), a sex by conditioning stimulus ($\mathbf{R} \times \mathbf{C}$) interaction would be said to exist. A first-order interaction ($\mathbf{R} \times \mathbf{C}$) is equivalent to differences between differences; a second-order interaction ($\mathbf{R} \times \mathbf{C} \times \mathbf{H}$) equivalent to differences between differences of differences; etc. (see example 8.8 below).

2. Equivalently, a first-order interaction ($\mathbf{R} \times \mathbf{C}$) can be thought of as a residual effect after the separate main effects of **R** and **C** have been taken out or allowed for. Thus, after any systematic (averaged over stimulus) sex difference in conditioning is allowed for, and any systematic (averaged over sex) difference in conditioning stimulus is also allowed for, if there remains any variation in the sex-stimulus cells, a sex by conditioning stimulus ($\mathbf{R} \times \mathbf{C}$) interaction would be said to exist. A second-order interaction ($\mathbf{R} \times \mathbf{C} \times \mathbf{H}$) would be said to exist if there was residual variation after the **R**, **C**, **H**, $\mathbf{R} \times \mathbf{C}$, $\mathbf{R} \times \mathbf{H}$, and $\mathbf{C} \times \mathbf{H}$ effects were removed, etc.

3. A third equivalent conception of an $\mathbf{R} \times \mathbf{C}$ interaction implied by either of the above is simply that the effect of **R** varies from one level of **C** to another (and conversely). Thus, a nonzero sex by conditioning stimulus interaction means (and is meant by): The effect of a given stimulus (relative to others) varies between sexes or depends upon which sex is under consideration. This, in turn, means that there is a *joint* effect of sex and stimulus over and above any separate (main) effect of the two variables. Equivalently, the effect of each is *conditional* on the other.

To index the size of an interaction, we use **f** defined in a way which is a generalization of the basic definition set forth in equations (8.2.1) and (8.2.2). First we return to the second conception of an $\mathbf{R} \times \mathbf{C}$ interaction above, where we spoke of a "residual effect" after the main effects of **R** and **C** have been taken out. Consider the cell defined by the **i**th level of **R** and the **j**th level of **C**, the **ij**th cell of the table, which contains in all **rc**

cells. That cell's population mean is $\mathbf{m_{ij}}$. Its value depends on (*a*) the main effect of $\mathbf{R_i}$, i.e., $\mathbf{m_{i\cdot}} - \mathbf{m}$, the departure of the population mean of level **i** of **R**, (*b*) the main effect of $\mathbf{C_j}$, i.e., $\mathbf{m_{\cdot j}} - \mathbf{m}$, the departure of the population mean of level **j** of **C**, (*c*) the value of **m**, and (*d*) the *interaction effect* for that cell, $\mathbf{x_{ij}}$, the quantity in which we are particularly interested. Simple algebra leads to the following definition of $\mathbf{x_{ij}}$ in terms of the cell mean ($\mathbf{m_{ij}}$), the main effect means $\mathbf{m_{i\cdot}}$, $\mathbf{m_{\cdot j}}$), and the total population mean ($\mathbf{m}$):

$$\mathbf{x_{ij}} = \mathbf{m_{ij}} - \mathbf{m_{i\cdot}} - \mathbf{m_{\cdot j}} + \mathbf{m}. \tag{8.3.5}$$

When a cell has $\mathbf{x_{ij}} = 0$, it has no interaction effect, i.e., its mean is accounted for by the $\mathbf{R_i}$ and $\mathbf{C_j}$ main effects and the total population mean. When all the **rc** cells have **x** values of zero, the $\mathbf{R} \times \mathbf{C}$ interaction is zero. Thus, the degree of *variability* of the **x** values about their (necessarily) zero mean is indicative of the size of the $\mathbf{R} \times \mathbf{C}$ interaction.

Thus, as a measure of the size of the interaction of the $\mathbf{R} \times \mathbf{C}$ factorial design, we use the standard deviation of the $\mathbf{x_{ij}}$ values in the **rc** cells. As an exact analogy to our (raw) measure of the size of a main effect, $\sigma_{\mathbf{m}}$ of formula (8.2.2), we find

$$\sigma_{\mathbf{x}} = \sqrt{\frac{\sum \mathbf{x_{ij}^2}}{\mathbf{rc}}}, \tag{8.3.6}$$

the square root of the mean of the squared interaction effect values for the **rc** cells.

To obtain a standardized ES measure of interaction, we proceed as before to divide by σ, the within-cell population standard deviation, to obtain **f**:

$$\mathbf{f} = \frac{\sigma_{\mathbf{x}}}{\sigma}. \tag{8.3.7}$$

The **f** for an interaction of formula (8.3.7) can be interpreted in the same way as throughout this chapter, as a measure of variability and hence size of (interaction) effects, whose mean is zero, standardized by the common within (cell) population standard deviation. Because it is the same measure, it can be understood:

1. in the framework which relates it to η and the proportion of variance of Section 8.2.2, as modified in terms of partial η for Case 2 in Section 8.3.3; or

2. By using the operational definitions of small, medium, and large **f** values of Section 8.2.3 (even though the discussion in these sections was

particularized in terms of the variability of means, rather than of interaction effects); or

3. By writing the alternate-hypothetical cell means and computing the **x** values and σ_x and **f** by formulas (8.3.5)–(8.3.7). (This latter procedure is illustrated in example 8.9 below.)

For the sake of simplicity of exposition, the above discussion has been of **f** for a two-way (first-order) interaction. The generalization of **f** for higher-order interactions is fairly straightforward. For example, given a three-way interaction, **R** × **C** × **H**, with **R** at **r** levels, **C** at **c** levels, and **H** at **h** levels, there are now **rch** cells. Consider the cell defined by the **i**th level of **R**, the **j**th level of **C**, and the **k**th level of **H**. Its interaction effect is

$$x_{ijk} = m_{ijk} - m_i - m_j - m_k - x_{ij} - x_{ik} - x_{jk} + 2m,$$

where the x_{ij}, x_{ik}, and x_{jk} are the two-way interaction effects as defined in formula (8.3.4). Analogous to formula (8.3.6), the raw variability measure is

$$\sigma_x = \sqrt{\frac{\sum x_{ijk}^2}{rch}}, \qquad (8.3.8)$$

i.e., the square root of the mean of the squared interaction effect values for the **rch** cells. It is then standardized by formula (8.3.7) to give **f**, the ES for a three-way interaction.

The number of degrees of freedom (**u**) for an interaction is the product of the **df**s of its constituent factors: (**r** − 1)(**c** − 1) for a two-way interaction, (**r** − 1)(**c** − 1)(**h** − 1) for a three-way interaction, etc.

For the reasons discussed in the preceding section on main effects, the test on interactions in factorial designs require that **n′** be used for table entry. Formula (8.3.4) is again used with the same denominator **df** as for the main effects and with **u** the appropriate **df** for the interaction.

In summary, power determination for interaction tests proceeds as follows: **u** is the **df** for the interaction and, together with the significance criterion **a**, determines the relevant power table. The table is then entered with **f**, which is determined by using one or more of the methods detailed above or by using the ES conventions, and **n′**, a function of the denominator **df** and **u** (8.3.4). The power value is then read from the table. Linear interpolation for **f**, **n**, and **u** (between tables) is used where necessary and provides a good approximation.

Illustrative Examples

8.7 Reconsider the experiment described in example 8.6, an inquiry in developmental social psychology in which the factors were sex of experimenter (**S** at **s** = 2 levels), age of subject (**A** at **a** = 3 levels), and instruction conditions (**C** at **c** = 4 levels), i.e., a 2 × 3 × 4 factorial design, and the dependent variable a measure of persuasibility. There are **n** = 5 subjects in each of the 24 cells of the design, a total **N** of 120, and the denominator **df** is 120 − 24 = 96. For convenience, we restate the specifications and resulting tabled power value for each of the main effect **F** tests:

S:	**a** = .05,	**u** = 1,	**f** = .10,	**n′** = .49;	power = .16
A:	**a** = .05,	**u** = 2,	**f** = .25,	**n′** = .33;	power = .59
C:	**a** = .05,	**u** = 3,	**f** = .40,	**n′** = .25;	power = .93

Consider first the interaction of sex of experimenter by age of subject (**S** × **A**), which is posited to be of medium size, i.e., **f** = .25, and the same significance criterion, **a** = .05, is to be used. Note that this interaction concerns the residuals in the 2 × 3 table which results when the 4 levels of **C** are collapsed. The **df** for this interaction is therefore **u** = (2 − 1)(3 − 1) = 2. All the effects in this fixed factorial design, including the **S** × **A** effect, use as their error term the within-cell mean square, hence the denominator **df**, as noted above, is 120 − 24 = 96. This latter value and **u** are used in formula (8.3.4) to determine **n′** for table entry: **n′** = 96/(2 + 1) + 1 = 33. The specifications for the power of the **S** × **A** effect are thus:

a = .05, **u** = 2, **f** = .25, **n′** = 33.

In Table 8.3.13 for **a** = .05 and **u** = 2, with row **n** = 33 and column **f** = .25, the power of the test is found as .59, a rather unimpressive value. Note that this is exactly the same value as was found for the **A** main effect, which is necessarily the case, since the specifications are the same. For **A**, we also used **a** = .05 and **f** = .25, and its **u** is also 2. Since **S** × **A** and **A** (as well as the other effects) also share the same denominator **df**, their **n′** values are also necessarily the same.

Let us also specify **a** = .05 and **f** = .25 for the **S** × **C** interaction. It is based on the 2 × 4 table which results when the three levels of **A** are collapsed, and its **u** is therefore (2 − 1)(4 − 1) = 3. With the same denominator **df** of 96, the **n′** for this effect is 96/(3 + 1) + 1 = 25. Thus,

a = .05, **u** = 3, **f** = .25, **n′** = 25,

and Table 8.3.14 (for **a** = .05, **u** = 3) gives at row **n** = 33 and column **f** = .25 the power value .53. For the specifications for **a** and **f** the power is even poorer than for the **S** × **A** interaction. This is because the increase in **u** results in a decrease in **n′**.

The **A** × **C** interaction is defined by the 3 × 4 table that results when the sex of experimenters is ignored, and its **u** is therefore (3 − 1)(4 − 1) = 6. For this **u** and denominator **df** = 96, the **n′** here is 96/(6 + 1) + 1 = 14.7. For the sake of comparability, we again posit **a** = .05 and **f** = .25. The specifications for the test of the **A** × **C** interaction, then, are:

$$\mathbf{a} = .05, \quad \mathbf{u} = 6, \quad \mathbf{f} = .25, \quad \mathbf{n}' = 14.7.$$

In Table 8.3.17 (**a** = .05, **u** = 6), column **f** = .25 gives power values of .39 at **n** = 14 and .42 at **n** = 15; linear interpolation gives power of .41 for **n′** = 14.7. Note that, although the specifications remain **a** = .05 and **f** = .25, since **u** is now 6, the resulting drop in **n′** has produced a reduction in power relative to the other two two-way interactions.

Finally, the three-way **S** × **A** × **C** interaction has **u** = (2 − 1)(3 − 1)(4 − 1) = 6, the same as for the **A** × **C** interaction, and thus the same **n′** = 96/(6 + 1) + 1 = 14.7. If we posit, as before, **a** = .05, and **f** = .25, the specifications are exactly the same as for the **A** × **C** interaction,

$$\mathbf{a} = .05, \quad \mathbf{u} = 6, \quad \mathbf{f} = .25, \quad \mathbf{n}' = 14.7,$$

and necessarily the same power of .41 is found (Table 8.3.17).

Because the **df** for interactions are products of the **df**s of their constituent main effect factors (e.g., for **A** × **C**, **u** = 2 × 3 = 6), the interactions in a factorial design will generally have larger **u** values than do the main effects, and, given the structure of the formula for **n′** (8.3.4), their **n′** values will generally be smaller than those for the main effects. This in turn means that, for any given size of effect (**f**) and significance criterion (**a**), the power of the interaction tests in a factorial design will, on the average, be smaller than that of main effects (excepting 2^K designs, where they will be the same). This principle is even more clearly illustrated in the next example.

8.8 Consider an **A** × **B** × **C** fixed factorial design, 3 × 4 × 5 (= 60 cells), with three observations in each cell, so that **N** = 60 × 3 = 180. The within-cell error term for the denominator of the **F** tests will thus have 180 − 60 = 120 **df**. To help the reader get a feel for the power of main effect and interaction tests in factorial design as a function of **f**, **a**, **u**, and the **n′** of formula (8.3.4), tabled power values for the **F** tests in this experiment are given in Table 8.3.34 for the conventional **f** values for small, medium, and large ES at **a** = .01, .05, and .10. Note that although this is a rather large experiment, for many combinations of the parameters, the power values are low. Study of the table shows that

1. Unless a large ES of **f** = .40 is posited, power is generally poor. Even at **f** = .40, when **a** = .01 governs the test, two of the two-way interactions have power less than .80, and for the triple interaction it is only .49. It seems clear that unless unusually large experiments are undertaken, tests of small effects have abysmally low power, and those for medium interaction effects for **u** > 4

have poor power even at **a** = .10.

2. For a medium ES of **f** = .25, only the main effect tests at **a** = .10 have power values that give better than two to one odds for rejecting the null hypothesis. At **a** = .05, power ranges from poor to hopeless, and at .01, not even the tests of main effects have power as large as .50.

TABLE 8.3.34

POWER AS A FUNCTION OF **f**, **a**, **u**, AND **n′** IN A 3 × 4 × 5 DESIGN WITH $\mathbf{n_c}$ = 3 AND DENOMINATOR **df** = 120

			f = .10			f = .25			f = .40		
Effect	u	n′	a = .01	.05	.10	.01	.05	.10	.01	.05	.10
A	2	41	05	15	25	45	70	80	93	98	99
B	3	31	04	13	22	38	63	75	90	97	99
C	4	25	03	12	21	33	58	70	86	96	98
A × B	6	18.1	03	10	18	26	51	64	80	93	97
A × C	8	14.3	02	09	17	23	46	59	75	91	95
B × C	12	10.2	02	08	16	18	39	52	66	86	92
A × B × C	24	5.8	02	08	14	10	29	42	49	74	83

3. For ESs no larger than what is conventionally defined as small (**f** = .10), there is little point in carrying out the experiment: even at the most lenient **a** = .10 criterion, the largest power value is .25.

4. At the popular **a** = .05 level, only at **f** = .40 are the power values high (excepting even here the .74 value for the **A** × **B** × **C** effect).

5. The table clearly exemplifies the principle of lower power values for interactions, progressively so as the order of the interaction increases (or, more exactly, as **u** increases). For example, only for **f** = .40 at **a** = .10 does the power value for **A** × **B** × **C** exceed .80.

The preparation and study of such tables in experimental planning and post hoc power analysis is strongly recommended. The reader is invited, as an exercise, to compute such a table for a 3 × 4 design with 15 observations per cell, and hence the same **N** = 180 as above. Comparison of this table with Table 8.3.34 should help clarify the implications of few cells (hence smaller **u**, larger denominator **df**, and larger **n′** values) to power.

Because of the relative infirmity of tests of interactions due to their often large **u**, the research planner should entertain the possibility of setting, a priori, larger **a** values for the interaction tests than for the tests of main effects, usually .10 rather than .05. The price paid in credibility when the null hypothesis for an interaction is rejected may well be worth the increase in power thus attained. This decision must, of course, be made on the basis not only of the design and ES parameters which obtain, but also with the substantive issues of the research kept in mind.

8.9 A psychologist designs an experiment in which he will study the effects of age (**R**) at $\mathbf{r} = 2$ levels, nature of contingency of reinforcement (**C**) at $\mathbf{c} = 4$ levels, and their interaction ($\mathbf{R} \times \mathbf{C}$) on a dependent learning variable. There are to be 12 subjects in each of the $\mathbf{rc} = 8$ cells, and $\mathbf{a} = .05$ throughout.

We will use this example to illustrate the direct specification of the alternate hypothesis and hence the ES. Assume that the area has been well studied and the psychologist has a "strong" theory, so that he can estimate the within-cell population standard deviation $\sigma = 8$, and further, he can state as an alternative to the overall null hypothesis specific hypothetical values for each of the eight cell's population means, the $\mathbf{m}_{ij}$. The latter then imply the **R** means ($\mathbf{m}_{i.}$), the **C** means ($\mathbf{m}_{.j}$), and the grand mean **m**. They are as follows:

	C_1	C_2	C_3	C_4	$m_{i.}$
R_1	41	34	30	27	33
R_2	33	24	22	29	27
$m_{.j}$	37	29	26	28	30 = m

These values, in raw form, comprise his ES for the effects of **R**, **C**, and $\mathbf{R} \times \mathbf{C}$. Their conversion to **f** values for the main effects is quite straightforward. Applying formula (8.2.2) for **R** and **C**,

$$\sigma_{m_R} = \sqrt{\frac{(33-30)^2 + (27-30)^2}{2}} = \sqrt{9} = 3,$$

and

$$\sigma_{m_C} = \sqrt{\frac{(37-30)^2 + (29-30)^2 + (26-30)^2 + (28-30)^2}{4}} = \sqrt{17.5} = 4.183.$$

When these are each standardized by dividing by the within-population $\sigma = 8$ [formula (8.2.1)], he finds

$$\mathbf{f}_R = 3/8 = .375$$

and

$$\mathbf{f}_C = 4.183/8 = .523.$$

For the $\mathbf{R} \times \mathbf{C}$ interaction ES, he finds the interaction effects for each cell using formula (8.3.4)

$$\mathbf{x}_{ij} = \mathbf{m}_{ij} - \mathbf{m}_{i.} - \mathbf{m}_{.j} + \mathbf{m}.$$

Thus,

$$\mathbf{x}_{11} = 41 - 33 - 37 + 30 = +1$$

$$\mathbf{x}_{12} = 34 - 33 - 29 + 30 = +2$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\mathbf{x}_{24} = 29 - 27 - 28 + 30 = +4$$

These $\mathbf{x}_{ij}$ values for the 2×4 table of means are

	C_1	C_2	C_3	C_4
R_1	+1	+2	+1	−4
R_2	−1	−2	−1	+4

Note that they are so defined that they must sum to zero in every row and column; these constraints are what result in the **df** for the $\mathbf{R} \times \mathbf{C}$ interaction being $\mathbf{u} = (\mathbf{r} - 1)(\mathbf{c} - 1)$; in this case, $\mathbf{u} = 3$.

Applying formula (8.3.6) to these values,

$$\sigma_x = \sqrt{\frac{\sum \mathbf{x}_{ij}^2}{\mathbf{rc}}} = \sqrt{\frac{(+1)^2 + (+2)^2 + (+1)^2 + \cdots + (+4)^2}{2(4)}}$$

$$= \sqrt{\frac{44}{8}} = 2.345.$$

Standardizing to find **f** [formula (8.3.7)],

$$\mathbf{f}_{R \times C} = \sigma_x / \sigma = 2.345/8 = .293.$$

Thus, his alternative-hypothetical cell population means, together with an estimate of σ, have provided an **f** for the $\mathbf{R} \times \mathbf{C}$ effect (as well as for the main effects).

One of the ways in which to understand interactions, described in the introduction to this section, was as differences among differences. This is readily illustrated for this problem. Return to the cell means and consider such quantities as $\mathbf{m}_{1j} - \mathbf{m}_{2j}$, i.e., the difference (with sign) between the means of $\mathbf{A}_1$ and $\mathbf{A}_2$ for each level of **C**. They are, respectively, (41 − 33 =) +8, (34 − 24 =) +10, +8, and −2. Were these four values (+8, +10, +8, and −2) all equal, there would be zero interaction. Calling these values $\mathbf{D}_j$ and their mean $\bar{\mathbf{D}}$ (here +6) for simplicity, σ_x can be found for a 2 × **c** table by

$$\sigma_x = \sqrt{\frac{\sum_{j=1}^{c} (\mathbf{D}_j - \bar{\mathbf{D}})^2}{4\mathbf{c}}}$$

$$= \sqrt{\frac{(+8-6)^2 + (+10-6)^2 + (+8-6)^2 + (-2-6)^2}{4(4)}}$$

$$= \sqrt{\frac{88}{16}} = 2.345,$$

as before.

Since there are 8 (= **rc**) cells with 12 subjects in each for a total **N** = 96, the denominator **df** for the **F** tests of the main effects and the interaction is 96 − 8 = 88. For the interaction test, **u** = (2 − 1)(4 − 1) = 3; therefore, the **n**′ for table entry from formula (8.3.4) is 88/(3 + 1) + 1 = 23. The specifications for the test on the **R** × **C** interaction are thus:

$$\mathbf{a} = .05, \quad \mathbf{u} = 3, \quad \mathbf{f} = .293, \quad \mathbf{n}' = 23.$$

In Table 8.3.14 (for **a** = .05, **u** = 3) at row **n**′ = 23, we find power at **f** = .25 to be .49 and at **f** = .30 to be .66. Linear interpolation for **f** = .293 gives the approximate power value of .64. The power for the main effects:

R: **a** = .05, **u** = 3, **f** = .375, **n**′ = 45, power = .94;

C: **a** = .05, **u** = 3, **f** = .523, **n**′ = 23, power = .99.

Power under these specifications for **R** and **C** is very good, but is only .64 for the interaction, despite the fact that its **f** of .293 is larger than a conventionally defined medium effect and that the experiment is fairly large. Since the interaction is likely to be the central issue in this experiment, the power of .64 is hardly adequate. To increase it, the experimenter should weigh the alternatives of increasing the size of the experiment or using the more modest **a** = .10 for the interaction test. If, for example, he increases the cell size from 12 to 17, the total **N** becomes 136, the denominator **df** = 136 − 8 = 128, and **n**′ for **R** × **C** is 128/(3 + 1) + 1 = 33. The specifications then are

$$\mathbf{a} = .05, \quad \mathbf{u} = 3, \quad \mathbf{f} = .293, \quad \mathbf{n}' = 33,$$

and power is found (by interpolation) to be .81. The size of the experiment must be increased by 42% to raise the power of the interaction test from .64 to .81. On the other hand, increasing the **a** to .10 for the experiment as originally planned, i.e., for

$$\mathbf{a} = .10, \quad \mathbf{u} = 3, \quad \mathbf{f} = .293, \quad \mathbf{n}' = 23,$$

power is found to be .75.

8.3.5 The Analysis of Covariance. With a simple conceptual adjustment of frame of reference, all the previous material in this chapter can be applied to power analysis in the analysis of covariance.

In the analysis of covariance (with a single covariate), each member of the population has, in addition to a value **Y** (the variable of interest or dependent variable) a value on another variable, **X**, called the concomitant or adjusting variable, or covariate. A covariance design is a procedure for statistically controlling for **X** by means of a regression adjustment so that one can study **Y** freed of that portion of its variance linearly associated with **X**. In addition to the assumptions of the analysis of variance, the method of covariance adjustment also assumes that the regression coefficients in the separate populations are equal. Detailed discussion of the analysis of covariance is beyond the scope of this treatment; the reader is referred to one of the standard texts: Blalock (1972), Winer (1971).

Instead of analyzing **Y**, the analysis of covariance analyzes **Y**′, a regression-adjusted or statistically controlled value, which is

$$\mathbf{Y}' = \mathbf{Y} - \mathbf{b}(\mathbf{X} - \bar{\mathbf{X}}), \tag{8.3.9}$$

where **b** is the (common) regression coefficient of **Y** on **X** in each of the populations and $\bar{\mathbf{X}}$ is the grand population mean of the concomitant variable. **Y**′ is also called a residual, since it is the departure of the **Y** value from the **YX** regression line common to the various populations.

The analysis of covariance is essentially the analysis of variance of the **Y**′ measures. Given this, if one reinterprets the preceding material in this chapter as referring to means and variances of the adjusted or residual **Y**′ values, it is all applicable to the analysis of covariance.

For example, the basic formula for **f** (8.2.1) is σ_m/σ. For covariance analysis, σ_m is the standard deviation of the **k** population's *adjusted* means of **Y**′, that is, **m**′, and σ is the (common) standard deviation of the **Y**′ values within the populations. The **d** measure of Section 8.2.1 is the difference between the largest and smallest of the **k** *adjusted* means divided by the within-population standard deviation of the **Y**′ values. The use and interpretation of η^2 as a proportion of variance and η as a correlation ratio now refers to **Y**′, the dependent variable **Y** freed from that portion of its variance linearly associated with **X**. And so on.

An academic point: In the analysis of covariance, the denominator **df** is reduced by one (due to the estimation of the regression coefficient **b**). This discrepancy from the denominator **df** on which the tabled power values are based is of no practical consequence in most applications, say when $(\mathbf{u}+1)(\mathbf{n}-1)$ is as large as 15 or 20.

The analysis of covariance can proceed with multiple covariates $\mathbf{X}_i$ ($\mathbf{i} = 1, 2, \ldots, \mathbf{p}$) as readily, in principle, as with one. The adjustment proceeds by multiple linear regression, so that

(8.3.10) $$\mathbf{Y}' = \mathbf{Y} - \mathbf{b}_1(\mathbf{X}_1 - \overline{\mathbf{X}}_1) - \mathbf{b}_2(\mathbf{X}_2 - \overline{\mathbf{X}}_2) - \cdots - \mathbf{b}_p(\mathbf{X}_p - \overline{\mathbf{X}}_p).$$

Whether **Y**′ comes about from one or several adjusting variables, it remains conceptually the same. The loss in denominator **df** is now **p** instead of 1, but unless **p** is large and **N** is small (say less than 40), the resulting overestimation of the tabled power values is not material.

The procedural emphasis should not be permitted to obscure the fact that the analysis of covariance designs when appropriately used yield greater power, in general, than analogous analysis of variance designs. This is fundamentally because the within-population σ of the *adjusted* **Y**′ variable will be smaller than σ of the unadjusted **Y** variable. Specifically, where **r** is the population coefficient between **X** and **Y**, $\sigma_{y}' = \sigma_y\sqrt{1-\mathbf{r}^2}$. Since σ is the denominator of **f** [formula (8.2.1)] and since the numerator undergoes no such systematic change (it may, indeed, increase), the *effective* **f** in an analysis of covariance will be larger than **f** in the analysis of variance of **Y**. This is true, of course, only for the proper use of the analysis of covariance, for discussion of which the reader is referred to the references cited above.

No illustrative examples are offered here because all of the eight examples which precede can be reconsidered in a covariance framework by merely assuming for each the existence of one or more relevant covariates. Each problem then proceeds with adjusted (**Y**′) values in place of the unadjusted (**Y**) values in which they are couched.

A very general approach to the analysis of covariance (and also the analysis of variance) is provided by multiple regression/correlation analysis, as described by Cohen and Cohen (1983). Some insight into this method and a treatment of its power-analytic procedures are given in Chapter 9.

8.4 Sample Size Tables

The sample size tables for this section are given on pages 381–389; the text follows on page 390.

Table 8.4.1

n to detect f by F test at a = .01
for u = 1, 2, 3, 4

u = 1

Power	f = .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	336	85	39	22	15	11	9	7	5	4	4	3
.50	1329	333	149	85	55	39	29	22	15	11	9	7
.70	1924	482	215	122	79	55	41	32	21	15	12	9
.80	2338	586	259	148	95	67	49	38	25	18	14	11
.90	2978	746	332	188	120	84	62	48	31	22	17	13
.95	3564	892	398	224	144	101	74	57	37	26	20	16
.99	4808	1203	536	302	194	136	100	77	50	35	26	21

u = 2

Power	f = .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	307	79	36	21	14	10	8	6	5	4	3	3
.50	1093	275	123	70	45	32	24	19	13	9	7	6
.70	1543	387	173	98	63	44	33	26	17	12	10	8
.80	1851	464	207	117	76	53	39	30	20	14	11	9
.90	2325	582	260	147	95	66	49	38	25	18	14	11
.95	2756	690	308	174	112	78	58	45	29	21	16	12
.99	3658	916	408	230	148	103	76	59	38	27	20	16

u = 3

Power	f = .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	278	71	32	19	13	9	7	6	4	3	3	2
.50	933	234	105	59	38	27	20	16	11	8	6	5
.70	1299	326	146	83	53	37	28	22	14	10	8	7
.80	1548	388	175	98	63	44	33	25	17	12	9	8
.90	1927	483	215	122	78	55	41	31	21	15	11	9
.95	2270	568	253	143	92	64	48	37	24	17	13	10
.99	2986	747	333	188	121	84	62	48	31	22	17	13

u = 4

Power	f = .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	253	64	29	17	12	8	7	5	4	3	3	2
.50	820	206	92	52	34	24	18	14	10	7	6	5
.70	1128	283	127	72	46	33	24	19	13	9	7	6
.80	1341	336	150	85	55	38	29	22	15	11	8	7
.90	1661	416	186	105	68	47	35	27	18	13	10	8
.95	1948	488	218	123	79	55	41	32	21	15	11	9
.99	2546	640	286	160	103	76	53	41	27	19	14	11

Table 8.4.2

n to detect f by F test at a = .01
for u = 5, 6, 8, 10

u = 5

Power	f: .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	233	59	27	16	11	8	6	5	4	3	2	2
.50	737	185	82	47	30	22	16	13	9	6	5	4
.70	1009	253	113	64	41	29	22	17	11	8	6	5
.80	1193	299	134	76	49	34	26	20	13	10	7	6
.90	1469	368	164	93	60	42	31	24	16	12	9	7
.95	1719	431	192	109	70	49	36	28	18	13	10	8
.99	2235	560	249	141	91	63	47	36	24	17	13	10

u = 6

Power	f: .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	218	55	25	15	10	7	6	5	3	3	2	2
.50	673	169	76	43	28	20	15	12	8	6	5	4
.70	917	230	103	58	38	27	20	15	10	8	6	5
.80	1080	271	121	68	44	31	23	18	12	9	7	6
.90	1326	332	148	84	54	38	28	22	14	10	8	6
.95	1547	388	173	98	63	44	33	25	17	12	9	7
.99	2003	502	224	126	81	57	42	33	21	15	11	9

u = 8

Power	f: .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	194	49	23	13	9	6	5	4	3	3	2	2
.50	580	146	65	37	24	17	13	10	7	5	4	3
.70	785	197	88	50	32	23	17	13	9	7	5	4
.80	918	230	103	58	38	27	20	15	10	8	6	5
.90	1122	281	126	71	46	32	24	19	12	9	7	6
.95	1303	327	146	83	53	37	28	22	14	10	8	6
.99	1676	420	187	106	68	48	36	27	18	13	10	8

u = 10

Power	f: .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	176	45	21	12	8	6	5	4	3	2	2	2
.50	515	129	58	33	21	15	12	9	6	5	4	3
.70	691	173	78	44	29	20	15	12	8	6	5	4
.80	810	203	91	51	33	23	18	14	9	7	5	4
.90	982	246	110	62	40	28	21	16	11	8	6	5
.95	1138	285	127	72	47	33	24	19	12	9	7	6
.99	1456	365	163	92	60	42	31	24	16	11	9	7

Table 8.4.3

n to detect f by F test at a = .01
for u = 12, 15, 24

u = 12

Power	f: .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	162	41	19	11	8	5	4	4	3	2	2	2
.50	467	117	53	30	20	14	10	8	6	4	3	3
.70	623	157	70	40	26	18	14	11	7	5	4	3
.80	726	182	82	46	30	21	16	12	8	6	5	4
.90	881	221	99	56	36	25	19	15	10	7	6	5
.95	1017	255	114	65	42	29	22	17	11	8	6	5
.99	1297	325	145	83	53	37	28	21	14	10	8	6

u = 15

Power	f: .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	147	37	17	10	7	5	4	3	2	2	2	--
.50	413	104	47	27	17	12	9	7	5	4	3	3
.70	548	138	62	35	23	16	12	10	6	5	4	3
.80	632	159	71	41	26	19	14	11	7	5	4	4
.90	769	193	86	49	32	22	17	13	9	6	5	4
.95	885	222	99	56	36	26	19	15	10	7	6	4
.99	1125	282	126	72	46	32	24	19	12	9	7	5

u = 24

Power	f: .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	118	30	14	8	6	4	3	3	2	2	--	--
.50	318	80	36	21	14	10	7	6	4	3	3	2
.70	417	105	47	27	17	12	9	7	5	4	3	3
.80	485	121	55	31	20	15	11	8	6	4	3	3
.90	578	145	65	37	24	17	13	10	7	5	4	3
.95	662	166	74	42	27	19	14	11	8	6	4	4
.99	831	209	92	53	34	24	18	14	9	7	5	4

Table 8.4.4

n to detect f by F test at a = .05
for u = 1, 2, 3, 4

u = 1

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	84	22	10	6	5	4	3	3	2	--	--	--
.50	769	193	86	49	32	22	17	13	9	7	5	4
.70	1235	310	138	78	50	35	26	20	13	10	7	6
.80	1571	393	175	99	64	45	33	26	17	12	9	7
.90	2102	526	234	132	85	59	44	34	22	16	12	9
.95	2600	651	290	163	105	73	54	42	27	19	14	11
.99	3675	920	409	231	148	103	76	58	38	27	20	15

u = 2

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	84	22	10	6	5	4	3	3	2	--	--	--
.50	662	166	74	42	27	19	15	11	8	6	5	4
.70	1028	258	115	65	42	29	22	17	11	8	6	5
.80	1286	322	144	81	52	36	27	21	14	10	8	6
.90	1682	421	188	106	68	48	35	27	18	13	10	8
.95	2060	515	230	130	83	58	43	33	22	15	12	9
.99	2855	714	318	179	115	80	59	46	29	21	16	12

u = 3

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	79	21	10	6	4	3	3	2	2	--	--	--
.50	577	145	65	37	24	16	13	10	7	5	4	3
.70	881	221	99	56	36	25	19	15	10	7	6	5
.80	1096	274	123	69	45	31	23	18	12	9	7	5
.90	1415	354	158	89	58	40	30	23	15	11	8	7
.95	1718	430	192	108	70	49	36	28	18	13	10	8
.99	2353	589	262	148	95	66	49	38	24	17	13	10

u = 4

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	74	19	9	6	4	3	2	2	--	--	--	--
.50	514	129	58	33	21	15	11	9	6	5	4	3
.70	776	195	87	49	32	22	17	13	9	6	5	4
.80	956	240	107	61	39	27	20	16	10	8	6	5
.90	1231	309	138	78	50	35	26	20	13	10	7	6
.95	1486	372	166	94	60	42	31	24	16	11	9	7
.99	2021	506	225	127	82	57	42	33	21	15	11	9

Table 8.4.5

n to detect f by F test at a = .05
for u = 5, 6, 8, 10

u = 5

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	69	18	9	5	4	3	2	2	--	--	--	--
.50	467	117	53	30	19	14	10	8	6	4	3	3
.70	698	175	78	44	29	20	15	12	8	6	5	4
.80	856	215	96	54	35	25	18	14	9	7	5	4
.90	1098	275	123	69	45	31	23	18	12	9	7	5
.95	1320	331	148	83	54	38	28	22	14	10	8	6
.99	1783	447	199	112	72	50	37	29	19	13	10	8

u = 6

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	66	17	8	5	4	3	2	2	--	--	--	--
.50	429	108	49	28	18	13	10	8	5	4	3	3
.70	638	160	72	41	26	18	14	11	7	5	4	4
.80	780	195	87	50	32	22	17	13	9	6	5	4
.90	995	250	112	63	41	29	21	16	11	8	6	5
.95	1192	299	133	75	49	34	25	20	13	9	7	6
.99	1604	402	179	101	65	46	34	26	17	12	9	7

u = 8

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	60	16	7	5	3	2	2	--	--	--	--	--
.50	374	94	42	24	16	11	8	7	5	4	3	2
.70	548	138	61	35	23	16	12	9	6	5	4	3
.80	669	168	75	42	27	19	14	11	8	6	4	4
.90	848	213	95	54	35	24	18	14	9	7	5	4
.95	1012	254	113	64	41	29	22	17	11	8	6	5
.99	1351	338	151	86	55	39	29	22	14	10	8	6

u = 10

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	55	14	7	4	3	2	2	--	--	--	--	--
.50	335	84	38	21	14	10	8	6	4	3	3	2
.70	488	123	55	31	20	14	11	8	6	4	3	3
.80	591	148	66	38	24	17	13	10	7	5	4	3
.90	747	187	84	48	31	22	16	13	8	6	5	4
.95	888	223	99	56	36	26	19	15	10	7	5	4
.99	1177	295	132	75	48	34	25	19	13	9	7	6

Table 8.4.6

n to detect f by F test at a = .05
for u = 12, 15, 24

u = 12

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	51	13	7	4	3	2	2	--	--	--	--	--
.50	306	77	35	20	13	9	7	6	4	3	3	2
.70	443	111	50	28	18	13	10	8	5	4	3	3
.80	534	134	60	34	22	16	12	9	6	5	4	3
.90	673	169	75	43	28	20	15	11	8	6	4	4
.95	796	200	89	51	33	23	17	13	9	6	5	4
.99	1052	264	118	67	43	30	22	17	11	8	6	5

u = 15

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	47	12	6	4	3	2	---	--	--	--	--	--
.50	272	69	31	18	12	8	6	5	4	3	2	2
.70	391	98	44	25	16	12	9	7	5	4	3	2
.80	471	118	53	30	20	14	10	8	6	4	3	3
.90	588	148	66	38	24	17	13	10	7	5	4	3
.95	697	175	78	44	29	20	15	12	8	6	4	4
.99	915	229	102	58	38	26	20	15	10	7	6	4

u = 24

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.10	38	10	5	3	2	---	---	--	--	--	--	--
.50	213	54	24	14	9	7	5	4	3	2	2	--
.70	303	76	34	20	13	9	7	5	4	3	2	2
.80	363	91	41	23	15	11	8	6	4	3	3	2
.90	457	115	51	29	19	13	10	8	5	4	3	3
.95	525	132	59	34	22	15	11	9	6	4	4	3
.99	680	171	76	44	28	20	15	11	8	6	4	4

Table 8.4.7

n to detect f by F test at a = .10
for u = 1, 2, 3, 4

u = 1

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	542	136	61	35	22	16	12	9	6	5	4	3
.70	942	236	105	60	38	27	20	15	10	7	6	5
.80	1237	310	138	78	50	35	26	20	13	9	7	6
.90	1713	429	191	108	69	48	36	27	18	13	10	8
.95	2165	542	241	136	87	61	45	35	22	16	12	9
.99	3155	789	351	198	127	88	65	50	32	23	17	13

u = 2

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	475	119	53	30	20	14	11	8	6	4	3	3
.70	797	200	89	50	32	23	17	13	9	6	5	4
.80	1029	258	115	65	41	29	22	17	11	8	6	5
.90	1395	349	156	88	57	40	29	23	15	11	8	6
.95	1738	435	194	109	70	49	36	28	18	13	10	8
.99	2475	619	276	155	100	70	51	33	21	15	11	9

u = 3

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	419	105	47	27	18	12	9	7	5	4	3	3
.70	690	173	77	43	28	20	15	11	8	6	4	4
.80	883	221	99	56	36	25	19	15	10	7	5	4
.90	1180	296	132	74	48	34	25	19	13	9	7	5
.95	1458	365	163	92	59	41	30	24	15	11	8	7
.99	2051	513	229	129	83	58	43	33	21	15	11	9

u = 4

Power	.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	376	95	43	24	16	11	9	7	5	4	3	3
.70	612	154	68	38	25	18	13	10	7	5	4	3
.80	773	193	87	49	32	22	17	13	9	6	5	4
.90	1031	258	115	65	42	29	22	17	11	8	6	5
.95	1267	317	141	80	51	36	27	21	13	10	7	6
.99	1768	443	197	111	71	50	37	28	19	13	10	8

Table 8.4.8

n to detect f by F test at a = .10
for u = 5, 6, 8, 10

u = 5

Power	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	343	86	39	22	14	10	8	6	4	3	3	2
.70	551	139	61	35	23	16	12	9	6	5	4	3
.80	693	174	77	44	28	20	15	12	8	6	4	4
.90	922	231	103.	58	37	26	20	15	10	7	6	4
.95	1128	283	126	71	46	32	24	18	12	9	7	5
.99	1564	392	175	98	63	44	33	25	16	12	9	7

u = 6

Power	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	317	80	36	20	13	9	7	6	4	3	3	2
.70	506	127	57	32	21	15	11	9	6	4	3	3
.80	635	159	71	40	26	18	14	11	7	5	4	3
.90	838	210	94	53	34	24	18	14	9	7	5	4
.95	1022	256	114	65	42	29	22	17	11	8	6	5
.99	1408	353	157	89	57	40	30	23	15	11	8	6

u = 8

Power	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	278	70	32	18	12	9	6	5	4	3	2	2
.70	436	110	49	28	18	13	10	8	5	4	3	3
.80	545	137	61	35	23	16	12	9	6	5	4	3
.90	717	180	80	46	29	21	15	12	8	6	4	4
.95	870	218	97	55	36	25	19	14	9	7	5	4
.99	1190	298	133	75	49	34	25	19	13	9	7	5

u = 10

Power	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	250	63	28	16	11	8	6	5	3	3	2	2
.70	390	98	44	25	16	11	9	7	5	4	3	2
.80	482	121	54	31	20	14	11	8	6	4	3	3
.90	633	159	71	40	26	18	14	11	7	5	4	3
.95	765	192	86	49	31	22	16	13	8	6	5	4
.99	1040	261	116	66	42	30	22	17	11	8	6	5

Table 8.4.9

n to detect f by F test at a = .10
for u = 12, 15, 24

u = 12

Power	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	229	58	26	15	10	7	5	4	3	2	2	2
.70	355	89	40	23	15	11	8	6	4	3	3	2
.80	437	110	49	28	18	13	10	8	5	4	3	3
.90	571	143	64	36	24	17	12	10	6	5	4	3
.95	688	173	77	44	28	20	15	11	8	5	4	4
.99	931	233	104	59	38	27	20	15	10	7	5	4

u = 15

Power	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	205	52	23	13	9	6	5	4	3	2	2	2
.70	315	79	35	20	13	9	7	6	4	3	2	2
.80	386	97	43	25	16	12	9	7	5	4	3	2
.90	502	126	56	32	21	15	11	9	6	4	3	3
.95	603	151	68	38	25	17	13	10	7	5	4	3
.99	812	203	91	51	33	23	17	13	9	6	5	4

u = 24

Power	f .05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
.50	161	41	18	11	7	5	4	3	2	2	--	--
.70	246	62	27	16	10	7	6	5	3	2	2	2
.80	298	75	34	19	12	9	7	5	4	3	2	2
.90	382	96	43	25	16	11	8	7	5	3	3	2
.95	456	114	52	30	19	13	10	8	5	4	3	3
.99	607	152	68	39	25	17	13	10	7	5	4	3

The tables in this section list values for the significance criterion (**a**), the numerator degrees of freedom (**u**), the ES to be detected (**f**), and the *desired power*. The required size per sample, **n**, may then be determined. The chief use of these tables is in the planning of experiments where they provide a basis for decisions about sample size requirements.

The 33 tables are laid out generally four to a table number, by **a** levels and successively tabled **u** values within each **a** level. The subtable for the required **a**, **u** combination is found and **f** and desired power are located. The same provisions for **a**, **u**, and **f** are made as for the tables in Section 8.3, as follows:

1. Significance Criterion, **a**. Table sets are provided for nondirectional **a** of .01, .05, and .10, each set made up of tables for varying values of **u**.

2. *Numerator Degrees of Freedom*, **u**. For each **a** level, tables are provided in succession for the 11 values of $u = 1$ (1) 6 (2) 12, 15, 24. Since the number of means to be compared is $k = u + 1$, the tables can be used directly for sets of means numbering $k = 2$ (1) 7 (2) 13, 16, and 25, and for interactions whose **df** equal the above 11 values of **u**. For missing values of **u** (7, 9, 11, etc.), linear interpolation between tables will yield adequate approximations to the desired **n**.

3. *Effect Size*, **f**. **f** was defined and interpreted for equal **n** in Sections 8.2, and generalized for unequal **n** in Section 8.3.2 and for interactions in Section 8.3.4. As in the power tables, provision is made in the sample size tables for the 12 values: .05 (.05) .40 (.10) .80. Conventional levels have been proposed (Section 8.2.3), as follows: small ES: $f = .10$, medium ES: $f = .25$, and large ES: $f = .40$. (No values of **n** less than 2 are given, since there would then be no within-population variance estimate from the data.)

To find **n** for a value of **f** not tabled, substitute in

$$n = \frac{n_{.05}}{400f^2} + 1, \qquad (8.4.1)$$

where $n_{.05}$ is the necessary sample size for the given **a**, **u**, and desired power at $f = .05$ (read from the table), and **f** is the nontabled ES. Round to the nearest integer.

4. *Desired Power*. Provision is made for desired power values of .10 (except at $a = .10$ where it would be meaningless), .50, .70, .80, .90, .95, .99. See 2.4.1 for the rationale for selecting such values for tabling, and particularly for a discussion of the proposal that .80 serve as a convention for desired power in the absence of another basis for a choice.

8.4.1 CASE 0: **k** MEANS WITH EQUAL **n**. The sample size tables were designed for this, the simplest case. Find the subtable for the significance criterion (**a**) and numerator **df** ($k - 1 = u$) which obtain and locate **f** and desired power, to determine **n**, the necessary size per each sample mean. For nontabled **f**, use the tables to find $n_{.05}$ and substitute in formula (8.4.1).

Illustrative Examples

8.10 Reconsider the educational experiment on the differential effectiveness of $k = 4$ teaching methods to equal sized samples of $n = 20$ (example 8.1). Using $a = .05$ as the significance criterion, and $f = .28$, it was found that power was approximately .53. Now we recast this as a problem in experimental planning, where we wish to determine the sample size necessary to achieve a specified power value, say .80. Initially, to illustrate the simplicity of the use of the sample size tables for tabled values of **f**, we change her specification of **f** to .25, our operational definition of a medium ES. Summarizing, the conditions for determining **n** for this test are

$$a = .05, \quad u = k - 1 = 3, \quad f = .25, \quad \text{power} = .80.$$

In the third subtable of Table 8.4.4 (for $a = .05$, $u = 3$) with column $f = .25$, and row power = .80, we find that we need $n = 45$ cases in each of the 4 method groups. Thus, slightly scaling down her ES from .28 to .25, she needs $4(45) = 180 = N$ to have .80 probability of a significant result at $a = .05$.

Since her **f** was originally .28, we illustrate the determination of **n** for this nontabled value, leaving the other specifications unchanged:

$$a = .05, \quad u = 3, \quad f = .28, \quad \text{power} = .80.$$

For nontabled **f**, we use formula (8.4.1). For $n_{.05}$, the sample size needed to detect $f = .05$ for $a = .05$, $u = 3$ with power = .80, we use the same subtable as above, the third subtable of Table 8.4.4 (for $a = .05$, $u = 3$) with column $f = .05$ and row power = .80 and find $n_{.05} = 1096$. Substituting in formula (8.4.1),

$$n = \frac{1096}{400(.28^2)} + 1 = \frac{1096}{31.36} + 1 = 35.9.$$

Thus, she would need 36 cases in each of the 4 groups to have power of .80 to detect $f = .28$ at $a = .05$. (This value of **n** is, as it should be, smaller than that which resulted when a smaller **f** of .25 was posited above.)

8.11 We reconsider the social psychiatric research of example 8.2, now as a problem in experimental planning. A pool of suitable in-patients is to be randomly assigned to $k = 3$ equal samples, and each subjected to a different microsocial system. Following this treatment, criterion measures will then be F-tested at $a = .01$. Temporarily, we revise the team's two proposed ES measures (the basis for which is described in example 8.2), $f = .229$ and .333, to a range of four tabled values: $f = .20, .25, .30, .35$. It is desired that power be .90 and we seek the n required for each of these specifications, which, in summary, are

$$a = .01, \quad u = k - 1 = 2, \quad f = \begin{cases} .20 \\ .25 \\ .30 \\ .35 \end{cases}, \quad \text{power} = .90.$$

We use the second subtable of Table 8.4.1 (for $a = .01$, $u = 2$) at row power = .90 and columns $f = .20, .25, .30$, and .35 and find the respective *per sample* n's of 147, 95, 66, and 49. Thus, for these conditions, an f of .20 requires three times as large an experiment as an f of .35. Note that in terms of proportion of variance, the respective η^2 for these values are .0385 and .1091 (Table 8.2.2).

Having illustrated the direct table look-up afforded by tabled f values, we turn to the actual f values posited by the two factions on the research team in the original example, .229 and .333. These nontabled values require the use of formula (8.4.1). The specifications are

$$a = .01, \quad u = 2, \quad f = \begin{cases} .229 \\ .333 \end{cases}, \quad \text{power} = .90.$$

For $n_{.05}$, the sample size needed to detect $f = .05$ for $a = .01$, $u = 2$, with power .90, we use the second subtable of Table 8.4.1 (for $a = .01$, $u = 2$) with column $f = .05$ and row power = .90 and find $n_{.05} = 2325$. Substituting it and $f = .229$ in formula (8.4.1),

$$n = \frac{2325}{400(.229^2)} + 1 = 111.8,$$

and for $f = .333$,

$$n = \frac{2325}{400(.333^2)} + 1 = 53.8.$$

Thus, if the "weak effect" faction ($f = .229$) is correct, samples of 112 cases are required, while if the "strong effect" faction ($f = .333$) is correct, only 54, less than half that number, are required per sample.

If they compromise by splitting the difference in n and use $(111 + 53)/2 =$

82 cases, we can solve formula (8.4.1) for f, the "detectable effect size,"[3] for given a, desired power, and n:

$$(8.4.2) \qquad f = \sqrt{\frac{n_{.05}}{400(n-1)}} = \sqrt{\frac{2325}{400(81)}} = .268.$$

The interpretation of this result is that for an F test at $a = .01$ of three means each based on 82 cases to have power of .90, the population ES must be $f = .268$. Since the relationship involved is not linear, splitting the difference in n does not split the difference on f. The latter would be $f = (.229 + .333)/2 = .281$. If the latter was the basis for compromise, the experiment would demand, applying formula (8.4.1) to these specifications,

$$n = \frac{2325}{400(.281^2)} + 1 = 74.6,$$

or 75 cases.

There is yet a third way of splitting the difference, i.e., between the .05 and .10 proportion of variance of criterion accounted for by experimental group membership, η^2. If the compromise is effected on this basis, $\eta^2 = (.05 + .10)/2 = .075$. Then, from formula (8.2.22),

$$f = \sqrt{\frac{.075}{1 - .075}} = .285.$$

Substituting this value of f with the $n_{.05} = 2325$ for these conditions in formula (8.4.1),

$$n = \frac{2325}{400(.285^2)} + 1 = 72.6,$$

or 73 cases, which hardly differs from the n demanded by averaging the f's (75). This will generally be the case unless the two f's are very widely separated.

8.4.2 CASE 2: k MEANS WITH UNEQUAL n. Sample size decisions for research planning in Case 2 offer no special problems. One must keep in mind

[3] The concept "detectable effect size" transcends its applications here. It is useful in *post hoc* power analysis, particularly in the assessment of failures to reject the null hypothesis and in summarizing the results of a series of experiments bearing on the same issue. See Cohen (1965, p. 100; 1970, p. 828).

that with unequal n_i, **f** is the standard deviation of the p_i-*weighted* standardized means, as described in Section 8.3.2. When the sample size tables are applied with the usual specifications, the **n** indicated in Case 2 is the *average* sample size of the **k** samples, i.e., $n = N/k$. Similarly, for nontabled **f**, the **n** found from formula (8.4.1) is the average sample size.

The unequal n_i case arises in research planning in various circumstances.

1. In political opinion, market research, or other surveys, where a total natural population is sampled and constitutent populations are of varying frequency, e.g., religious affiliations (as illustrated in Section 8.3.2), socioeconomic categories, etc. (See example 8.12 below.).
2. In experiments where one or more samples of fixed size are to be used, and the size of one or more samples is open to the determination of the experimenter. For example, scheduling problems may dictate that a control sample is to have 50 cases, but the sample sizes of two experimental groups can be determined using considerations of desired power.
3. In some experiments, it may be desired that a reference or control sample have larger **n** than the other $k-1$ samples. (See example 8.12 below.)

In each of these circumstances, the average **n** which is read from the tables [or computed from formula (8.4.1)] is multiplied by **k** to yield the total **N**.

Illustrative Examples

8.12 To illustrate Case 1 in surveys of natural populations, return to example 8.3, where a political science class designs an opinion survey of college students on government centralism. A source of variance to be studied is the academic areas of respondents of which there are 6 ($=k$). The **f** for the anticipated unequal n_i is posited at .15, and $a = .05$. Now, instead of treating this as a completed or committed experiment (where total **N** was set at 300 and power then found to be .48), let us ask what **N** is required to attain power of .80. The specifications are

$$a = .05, \quad u = k - 1 = 5, \quad f = .15, \quad \text{power} = .80.$$

In the first subtable of Table 8.4.5 (for $a = .05$, $u = 5$) at column $f = 15$ and row power = .80, $n = 96$. This is the average size necessary for the 6 academic area samples. The quantity we need is the total sample size, $N = 6(96) = 576$.

Example 8.3 went on to consider the effect on power of a reduction of **k** from 6 to 3 more broadly defined academic areas. Paralleling this, we determine **N** needed for $k = 3$, keeping the other specifications unchanged:

$$a = .05, \quad u = k - 1 = 2, \quad f = .15, \quad \text{power} = .80.$$

From the second subtable of Table 8.4.4 (for $a = .05$, $u = 2$) for column $f = .15$, row power = .80, we find $n = 144$, so that $N = 3(144) = 432$. Note that going from 6 to 3 groups results here in a 25% reduction of the **N** demanded (from 576 to 432). Of course, we assumed **f** to remain the same, which would probably not be the case.

8.13 A psychophysiologist is planning an experiment in which he will study the effect of two drugs (A and B) on neural regeneration relative to a control (C). He plans that $n_A = n_B$ (which we call n_E) but n_C is to be 40% larger, i.e., $n_C = 1.4n_E$. He posits that the three within-population-standardized mean differences will be $(m_A - m) = -.5$, $(m_B - m) = +.5$, and $(m_C - m) = 0$, that $a = .05$, and he wishes power to be .90. To determine the necessary sample size, he must first find the **f** implied by his alternate-hypothetical means. His total sample size is

$$N = n_E + n_E + 1.4n_E = 3.4n_E,$$

so

$$p_A = p_B = \frac{n_E}{N} = \frac{n_E}{3.4n_E} = .294$$

and

$$p_C = \frac{1.4n_E}{N} = \frac{1.4n_E}{3.4n_E} = .412.$$

Combining formulas (8.3.1), (8.3.2), and (8.2.1),[4]

$$(8.4.3) \qquad f = \sqrt{\sum p_i \left(\frac{m_i - m}{\sigma}\right)^2}$$

$$= \sqrt{.294(-.5^2) + .294(+.5^2) + .412(0^2)} = \sqrt{.1470} = .38.$$

Collecting the specifications,

$$a = .05, \quad u = k - 1 = 2, \quad f = .38, \quad \text{power} = .90.$$

[4] Although the means are equally spaced, we cannot use the **d** procedures of Section 8.2.1, which are predicated on equal **n**.

Since **f** is not tabled, we proceed to find the average **n** by formula (8.4.1), which calls for $\mathbf{n}_{.05}$, the **n** required for these specifications of **a**, **u**, and power when $\mathbf{f} = .05$. In the second subtable of Table 8.4.4, $\mathbf{a} = .05$ and $\mathbf{u} = 2$, row power = .90, and $\mathbf{f} = .05$, $\mathbf{n}_{.05} = 1682$. Applying formula (8.4.1),

$$\mathbf{n} = \frac{1682}{400(.38^2)} + 1 = 30.1.$$

But this **n** is for Case 1, the *average* **n** per sample. The total $\mathbf{N} = 3(30.1) = 90.3$. The sample sizes are unequal portions of this, as specified: The sample size of groups A and B are each .294(90.3) = 27 and of group C is .412(90.3) = 37. Thus, with sample sizes respectively for A, B, and C of 27, 27, and 37, he will have a .90 probability that his **F** test on the 3 sample means will meet the .05 significance criterion, given that $\mathbf{f} = .38$.

8.4.3 CASES 2 AND 3: FIXED MAIN AND INTERACTION EFFECTS IN FACTORIAL AND COMPLEX DESIGNS. In factorial design, the power values of tests of both main and interaction effects are determined by the design's denominator **df**, which in turn depends upon a single given cell sample size ($\mathbf{n}_c$). It is therefore convenient to present sample size determination for all the effects together for any given design. (In other complex designs, i.e., those with more than one source of nonerror variance, the same methods apply, although there may be different denominator **df**s for different effects.) The reader is referred to Sections 8.3.3 and 8.3.4 for discussions of interaction effects and the interpretation of η and η^2 as partial values.

The procedure for using the tables to determine the sample size required by an effect is essentially the same as for Cases 0 and 1. The sample size table (for specified **a** and **u**) is entered with **f** and the desired power, and the **n** is read from the table. However, this **n** must be understood as the **n′** of formula (8.3.4), a function of the denominator **df** and the **df** for the effect, **u**. The *cell* sample size implied by the **n′** value read from the table is then found from

$$\mathbf{n}_c = \frac{(\mathbf{n}' - 1)(\mathbf{u} + 1)}{\text{number of cells}} + 1, \tag{8.4.4}$$

where **u** is the **df** for the effect being analyzed, and "number of cells" is the number of (the highest order of) cells in the analysis, e.g., for all main and interaction effects in an $\mathbf{R} \times \mathbf{C} \times \mathbf{H}$ design it is **rch**. We assume throughout that all cells have the same $\mathbf{n}_c$. The $\mathbf{n}_c$ thus computed need not be an integer. It is therefore rounded up to the next higher integer (or down, if it is very close to the lower integer) to determine the cell sample size that must actually be employed. Multiplying this integral $\mathbf{n}_c$ value by the number of cells in the design then gives the actual total **N** required by the specifications for the effect in question.

When **f** is not a tabled value, one proceeds as in Cases 0 and 1 to find **n** by formula (8.4.1). This is again **n′**, and one proceeds as above to determine $\mathbf{n}_c$ and **N**.

Since the tests of the various effects in a factorial (or other complex) design will demand different **N**s, these must then be resolved into a single **N** which will then be used in the experiment.

Illustrative Examples

8.14 Reconsider example 8.6, now as a problem in sample size determination to achieve specified power. The experiment is concerned with the effects on persuasibility in elementary school boys of sex of experimenter (**S**), age of subject (**A**), and instruction conditions (**C**), in respectively a $2 \times 3 \times 4$ (= 24 cells) factorial design. The ES posited for the three main effects are $\mathbf{f}_S = .10$, $\mathbf{f}_A = .25$ and $\mathbf{f}_C = .40$ and for all interaction tests, $\mathbf{f} = .25$; all the tests are to be performed at $\mathbf{a} = .05$. Assume that power of .80 is desired for all of the tests, subject to reconsideration and reconciliation of the differing **N**'s which will result.

For the **S** effect, the specifications are thus:

$$\mathbf{a} = .05, \quad \mathbf{u} = 2 - 1 = 1, \quad \mathbf{f} = .10, \quad \text{power} = .80.$$

In the first subtable of Table 8.4.4 for $\mathbf{a} = .05$, $\mathbf{u} = 1$, with column $\mathbf{f} = .10$ and power = .80, we find the value 394. Treating it as **n′**, we then find from formula (8.4.4) that the cell sample size implied by **n′** is

$$\mathbf{n}_c = \frac{(394 - 1)(1 + 1)}{24} + 1 = (33.75) = 34,$$

and the actual total **N** required for the **S** effect by these specifications is 24(34) = 816 (!). Although conceivable, it seems unlikely that an experiment of this size would be attempted. Note that $\mathbf{f} = .10$ operationally defines a small ES, and we have seen in previous chapters that to have power of .80 to detect small ES requires very large sample sizes. This virtually restricts such attempts to large scale survey research of the type used in political polling and to sociological, market, and economic research.

Consider now the **N** demanded by the specifications for the age effect, which are

$$\mathbf{a} = .05, \quad \mathbf{u} = 3 - 1 = 2, \quad \mathbf{f} = .25, \quad \text{power} = .80.$$

In the second subtable of Table 8.4.4, for $\mathbf{a} = .05$ and $\mathbf{u} = 2$, with column

f = .25, and row power = .80, we find the **n** (= **n**′) value of 52. Substituting in (8.4.4), n_c = (52 − 1)(2 + 1)/24 + 1 = (7.38 =) 8, hence the actual total **N** = 24(8) = 192. This more modest **n** demand is primarily due to positing **f** = .25 (medium ES).

Finally, we find **n** required for the test on **C**, as specified:

$$a = .05, \quad u = 4 - 1 = 3, \quad f = .40, \quad \text{power} = .80.$$

The third subtable of Table 8.4.4 (for **a** = .05, **u** = 3) at **f** = .40, power = .80, yields the value 18 for **n** (= **n**′). n_c = (18 − 1)(3 + 1)/24 + 1 = (3.8 =) 4, so the total **N** required is 24(4) = 96. This relatively small required **N** is primarily a consequence of positing **f** = .40, a large ES.

Taking stock at this point, the three tests of the main effects, of varying specifications, have led to varying **N** demands of 816 for **S**, 192 for **A**, and 96 for **C**.

Turning now to the tests of the interactions, they all share the same **a** = .05, **f** = .25, and the power desired specified at .80. They differ only in their **u** values, but this means that they will differ in **n**′ and therefore **N**:

For **S** × **A**, **u** = (2 − 1)(3 − 1) = 2. The specifications are the same as for the **A** main effect (**a** = .05, **u** = 2, **f** = .25, power = .80), so the results are the same: eight cases per cell, hence **N** = 192.

For **S** × **C**, **u** = (2 − 1)(4 − 1) = 3. From the third subtable of Table 8.4.4 (**a** = .05, **u** = 3), for power = .80 when **f** = .25, the value **n**′ = 45 is found. Formula (8.4.4) then gives n_c = (45 − 1)(3 + 1)/24 + 1) = (8.33 =) 9, and **N** = 24(9) = 216.

For **A** × **C**, **u** = (3 − 1)(4 − 1) = 6. The second subtable of Table 8.4.5 (**a** = .05, **u** = 6) gives **n**′ = 32 for power = .80, **f** = .25. Formula (8.4.4) then gives n_c = (32 − 1)(6 + 1)/24 + 1 = (10.04 =) 10 (We round down here since 10.04 is only trivially larger than 10.) **N** is therefore 24(10) = 240.

Finally, for the test of the **S** × **A** × **C** interaction effect, **u** = (2 − 1) (3 − 1)(4 − 1) = 6, and the specifications are the same as for **A** × **C**, therefore n_c = 10 and **N** = 240.

We have thus had an array of **N** values demanded by the three main and four interaction effects ranging from 96 to 816, and some choice must be made. Table 8.4.10 summarizes the specifications and resulting sample size demands for the seven tests of this 2 × 3 × 4 factorial design. Surveying the results of this analysis, the researcher planning this experiment may reason as follows:

The central issues in this research are the interactions, so the fact that adequate power for the small **S** effect is beyond practical reach (816 cases in a manipulative experiment is virtually unheard of) is not fatal. If an experiment as large as **N** = 240 can be mounted, power of at least .80 at **a** = .05 can be attained for the ES values specified. The actual power values for all

the tests are then determined by the methods of Sections 8.3.3 and 8.3.4. They turn out to be: **S** .31, **A** .91, **C** >.995, **S** × **A** .92, **S** × **C** .88, **A** × **C** .80, and **S** × **A** × **C** .80.

TABLE 8.4.10

SAMPLE SIZE DEMANDS FOR THE MAIN AND INTERACTION EFFECTS IN THE **S** × **A** × **C** (2 × 3 × 4) FACTORIAL DESIGN

	Specifications					
Effect	a	u	f	Power	n_c	N
S	.05	1	.10	.80	34	816
A	.05	2	.25	.80	8	192
C	.05	3	.40	.80	4	96
S × **A**	.05	2	.25	.80	8	192
S × **C**	.05	3	.25	.80	9	216
A × **C**	.05	6	.25	.80	10	240
S × **A** × **C**	.05	6	.25	.80	10	240

Alternatively, it may well be the case that **N** = 240 exceeds the resources of the researcher, but after studying Table 8.4.10 he decides that he can (barely) manage eight cases per cell and **N** = 192; this will provide adequate power for **A**, **C**, and **S** × **A** (**S** is hopeless, anyway). The actual power values with **N** = 192 for the tests of the interactions are then determined to be: **S** × **A** .84, **S** × **C** .79, **A** × **C** .68, and **S** × **A** × **C** .68. The planner may be willing to settle for these values and proceed with **N** = 192.

On the other hand, we may judge that the two-to-one odds for rejection in the **F** tests of the **A** × **C** and **S** × **A** × **C** interactions are not good enough. He may be willing to decide, a priori, that he is prepared to test these interactions at **a** = .10. Note that he need not shift to **a** = .10 for the other tests. He is simply prepared to offer a somewhat less credible rejection of these two null hypotheses if it should turn out that the increase in power is sufficient to make it worthwhile. These tests will thus have the same specifications: **a** = .10, **u** = 6, **f** = .25, and, since **N** = 192, denominator **df** = 192 − 24 = 168, and **n**′ = 168/(6 + 1) + 1 = 25. Looking up **n** = 25 at **f** = .25 in Table 8.3.28 (for **a** = .10, **u** = 6), he finds power = .78. He may then consider whether he prefers power of .68 at **a** = .05 or power of .78 at **a** = .10 for these two tests, a not very happy pair of alternatives. (A factor in his decision may be his judgment as to whether **f** = .25 is a possibly overoptimistic estimation of the true ES. If so, he had better opt for the **a** = .10 alternative since, at **a** = .05, power would be less than .68).

There is another device available in research planning to bring sample size demands into conformity with available resources, already illustrated in problem 8.3. One should consider dropping the number of levels of a research factor in order to reduce the size of **u**, particularly in interactions. In this illustration, if only two age groups are used, **u** = 3 for **A** × **C** and **S** × **A** × **C.** For **N** = 192, now in 2 × 2 × 4 = 16 cells (hence, $\mathbf{n}_c = 12$), the denominator **df** will be 192 − 16 = 176, and **n**′ will be 176/(3 + 1) = 1 = 45. For **a** = .05 and **u** = 3, Table 8.3.14 gives power = .81 at **f** = .25 for **n** = 45. This appears to be the preferred resolution of the problem in this illustration. In other circumstances an entire research factor may be dropped in the interests of increasing power or decreasing sample size demand for the remainder of the experiment.

8.15 We return to example 8.9 which described a learning experiment of the effects of age (**R**) at **r** = 2 levels and contingency of reinforcement (**C**) at **c** = 4 levels on a measure of learning, so that there are 2 × 4 = 8 cells. Although **f** may be specified by using the operational definition conventions, example 8.9 illustrated how **f** values for the main effects and interaction are arrived at by positing values for the alternate-hypothetical cell means and within-population σ and computing them from these values. We found there that **f** for **R** was .375, for **C** .523, and for **R** × **C** .293. The problem is now recast into one in which sample size is to be determined, given the desired power and the other specifications. Assume initially that all three tests are to be performed at **a** = .05 and that the power desired is at least .80.

For the test of the **R** (age) effect, the specification summary is thus:

$$\mathbf{a} = .05, \qquad \mathbf{u} = \mathbf{r} - 1 = 1, \qquad \mathbf{f} = .375, \qquad \text{power} = .80.$$

Since **f** = .375 is not a tabled value, we proceed by means of formulas (8.4.1) and (8.4.4). In the first subtable of Table 8.4.4 (**a** = .05, **u** = 1), at power = .80, the value at **f** = .05 is 1571. Thus, from (8.4.1),

$$\mathbf{n}' = \frac{1571}{400(.375^2)} + 1 = 28.93,$$

and then applying formula (8.4.4),

$$\mathbf{n}_c = \frac{(28.93 - 1)(1 + 1)}{8} + 1 = (7.98 =)\ 8,$$

so that each of the eight cells will have eight cases, and **N** = 64 cases are required for the test of the **R** effect.

For the test of the reinforcement contingency (**C**) effect, the specifications are:

$$\mathbf{a} = .05, \qquad \mathbf{u} = \mathbf{c} - 1 = 3, \qquad \mathbf{f} = .523, \qquad \text{power} = .80.$$

The third subtable of Table 8.4.4 (**a** = .05, **u** = 3), gives $\mathbf{n}_{.05} = 1096$ for power = .80. Formula (8.4.1) then gives, for **f** = .523,

$$\mathbf{n}' = \frac{1096}{400(.523^2)} + 1 = 11.02$$

and formula (8.4.4) gives

$$\mathbf{n}_c = \frac{(11.02 - 1)(3 + 1)}{8} + 1 = (6.01 =)\ 6,$$

so that **N** = 8 × 6 = 48, a substantially smaller demand for the test of the **C** effect.

The specifications for the test of the **R** × **C** interaction effect are:

$$\mathbf{a} = .05, \qquad \mathbf{u} = (\mathbf{r} - 1)(\mathbf{c} - 1) = 3, \qquad \mathbf{f} = .293, \qquad \text{power} = .80,$$

and, since **a**, **u**, and power are the same as for the **R** main effect, the $\mathbf{n}_{.05} = 1096$ is the same. For **f** = .293,

$$\mathbf{n}' = \frac{1096}{400(.293^2)} + 1 = 32.92,$$

and

$$\mathbf{n}_c = \frac{(32.92 - 1)(3 + 1)}{8} + 1 = (16.96 =)\ 17$$

so **N** = 8 × 17 = 136 for the **R** × **C** test.

So again, as will so often be the case for interactions, the sample size demand is large relative to those for the main effects. If the experimenter is prepared to mount that large an experiment, power for testing the interaction effect will be .80, and it will be much better than that for the main effects:

R: **a** = .05, **u** = 1, **f** = .375, **n**′ = (136 − 8)/(1 + 1) + 1 = 65.

From Table 8.3.12, power = .99.

C: **a** = .05, **u** = 3, **f** = .523, **n**′ = (136 − 8)/(3 + 1) + 1 = 33.

From Table 8.3.14, power > .995.

If the experimenter finds **N** = 136 a larger experiment than he can manage, he may investigate the consequence to the **N** required by switching to an **a** = .10 criterion for the **R** × **C** test. For this change in the specifications, $\mathbf{n}_{.05}$ for **a** = .10, **u** = 3 (third subtable of Table 8.4.7) is 883, **n**′ = 26.71, $\mathbf{n}_c = 14$ and **N** = 112.

As another possibility, he may retain **a** = .05, but settle for power = .70 for the **R** × **C** test. From Table 8.4.4 for **a** = .05, **u** = 3, $\mathbf{n}_{.05}$ is found to be

881, so $\mathbf{n}'$ is computed as 26.66, $\mathbf{n}_c$ as 14 and $\mathbf{N} = 112$. Thus, for the reduction in **N** from 136 to 112, he may either use the lenient $\mathbf{a} = .10$ criterion with power = .80, or the conventional $\mathbf{a} = .05$ but with power = .70.

Finally, as in the preceding problem, he may consider giving up one of the reinforcement conditions so that there are only $2 \times 3 = 6$ cells and the **u** for $\mathbf{R} \times \mathbf{C}$ is reduced to $(2-1)(3-1) = 2$. If the choice of which condition to omit may be made on purely statistical grounds, the table of alternate-hypothetical population means presented in problem 8.9 above suggests that $\mathbf{C}_3$ is the best candidate. Note that the omission of the means for $\mathbf{C}_3$ will change all three **f** values. The **f** for $\mathbf{R} \times \mathbf{C}$ increases to .328 (and is slightly decreased for the main effects). For the revised 2×3 design, then, the specifications for $\mathbf{R} \times \mathbf{C}$ are:

$$\mathbf{a} = .05, \quad \mathbf{u} = 2, \quad \mathbf{f} = .328, \quad \text{power} = .80,$$

and via formulas (8.4.1) and (8.4.4), $\mathbf{n}_c$ is found to be 16 and $\mathbf{N} = 6 \times 16 = 96$. (The reader may wish to check the above as an exercise.) Thus, by removing the condition that makes the least contribution to the interaction, its **f** is increased (from .293 to .328), its **u** is decreased, and the result is that for $\mathbf{a} = .05$ and power = .80, 96 rather than 136 cases are required. The experimenter might well decide to follow this course.

This and the preceding problem tell a morality tale about research design. The possibility of studying many issues within a single experiment, so well described in the standard textbooks on experimental design and the analysis of variance, should be accompanied by a warning that the power of the resulting tests will be inadequate unless **N** is (usually unrealistically) large or the ESs are (also usually unrealistically) large. Recall that this principle is not restricted to factorial or other complex designs; a simple one-way analysis of variance on **k** groups will, unless **f** is large, require relatively large **N** (as illustrated in problem 8.3). Consider the standard conditions $\mathbf{a} = .05$, $\mathbf{f} = .25$ (medium ES), and desired power = .80 for a one-way design with **k** groups. Table 8.4.11 shows now the required **n** per group and total **N** (= **nk**) vary as **k** increases (the **n** values are simply read from Tables 8.4.4–8.4.6). Although the required sample size *per group* decreases as **k** increases, the total **N** increases with **k**. Although for a medium ES 150 subjects provide adequate power to appraise two or three treatments, that number is not sufficient for six or seven. The reader might find it instructive to construct and study tables like 8.4.11 for other values of **f** and **a**.

TABLE 8.4.11

n PER GROUP AND TOTAL **N** AS A FUNCTION OF **k** FOR **k** GROUPS UNDER THE CONDITIONS $\mathbf{a} = .05$ AND POWER = .80 FOR $\mathbf{f} = .25$

k	u	n	N
2	1	64	128
3	2	52	156
4	3	45	180
5	4	39	195
6	5	35	210
7	6	32	224
9	8	27	243
11	10	24	264
13	12	22	286
16	15	20	320
25	24	15	375

8.4.5 THE ANALYSIS OF COVARIANCE. As was discussed in the section on the use of the power tables in the analysis of covariance (8.3.5), no special procedural change takes place from analogous analysis of variance designs. What changes is the conception of the dependent variable, which becomes **Y′**, a regression-adjusted or statistically controlled value [defined in formula (8.3.9)], whose use may result in a larger ES than the use of the unadjusted **Y**. Population means, variances, ranges, etc., now merely refer to this adjusted variable in place of the unadjusted variable of the analysis of variance. For more detail, see Section 8.3.5. See also the alternative approach to data-analytic problems of this kind by means of multiple regression/correlation analysis in Chapter 9.

Thus, sample size estimation in the analysis of covariance proceeds in exactly the same way as in analogous analysis of variance designs.

8.5 THE USE OF THE TABLES FOR SIGNIFICANCE TESTING

8.5.1 INTRODUCTION. As is the case in most of the chapters in this handbook, provision for facilitating significance testing has been made in the power tables as a convenience to the reader. While power analysis is primarily relevant to experimental planning and has as an important parameter the alternative-hypothetical population ES, once the research data are collected, attention turns to the assessment of the null hypothesis in the light of the data (Cohen, 1973). (See Section 1.5, and for some of the advantages of the corollary approach in **t** tests, Section 2.5.)

Because of the discrepancy between the actual denominator **df** in a factorial or other complex design and the one-way design (Cases 0 and 1) assumed in the construction of the tables, it does not pay to undertake the adjustments that would be necessary to use the tabled values of $\mathbf{F}_c$ for significance testing in Cases 2 and 3, since **F** tables are widely available in statistical textbooks and specialized collections (e.g., Owen, 1962). Accordingly, we do not discuss or exemplify the use of the $\mathbf{F}_c$ values in the power tables in this handbook for significance testing of fixed main effects or interactions (Cases 2 and 3).

For significance testing, the function of the data of interest to us in the Case 0 and 1 applications of this chapter is the **F** ratio for the relevant null hypothesis which is found in the sample, $\mathbf{F}_s$.

In each power table (8.3) for a given significance criterion **a** and numerator **df, u**, the second column contains $\mathbf{F}_c$, the minimum **F** necessary for significance at the **a** level for that **u**. The $\mathbf{F}_c$ values vary with **n**, the relevant sample size. Significance testing proceeds by simply comparing the computed $\mathbf{F}_s$ with the tabled $\mathbf{F}_c$.

8.5.2 SIGNIFICANCE TESTING IN CASE 0: **k** MEANS WITH EQUAL **n**. Find the power table for the significance criterion (**a**) and numerator **df**, $\mathbf{u} = \mathbf{k} - 1$, which obtain. Enter with **n**, the size per sample mean, and read out $\mathbf{F}_c$. If the computed $\mathbf{F}_s$ equals or exceeds the tabulated $\mathbf{F}_c$, the null hypothesis is rejected.

Illustrative Examples

8.16 Assume that the educational experiment described in 8.1 has been performed: a comparison (at $\mathbf{a} = .05$) of the differential effectiveness of $\mathbf{k} = 4$ teaching methods, for each of which there is a random sample of $\mathbf{n} = 20$. Whatever the history of the planning of this experiment, including most particularly the anticipated ES ($\mathbf{f} = .280$), what is *now* relevant is the **F** value (between groups mean square/within groups mean square) computed from the 4(20) = 80 achievement scores found in the completed experiment, $\mathbf{F}_s$. Assume $\mathbf{F}_s$ is found to equal 2.316. Thus, the specifications for the significance test are

$$\mathbf{a} = .05, \quad \mathbf{u} = \mathbf{k} - 1 = 3, \quad \mathbf{n} = 20, \quad \mathbf{F}_s = 2.316.$$

To determine the significance status of the results, checking column $\mathbf{F}_c$ of Table 8.3.14 ($\mathbf{a} = .05$, $\mathbf{u} = 3$) for $\mathbf{n} = 20$ gives $\mathbf{F}_c = 2.725$. Since the computed $\mathbf{F}_s$ of 2.316 is smaller than the criterion value, the results are not significant at $\mathbf{a} = .05$, i.e., the data do not warrant the conclusion that the population achievement means of the four teaching methods differ.

8.17 In example 8.2, a power analysis of an experiment in social psychiatry was described in which $\mathbf{k} = 3$ equal samples of $\mathbf{n} = 200$ each were subjected to different microsocial systems. Consider the experiment completed and the data analyzed. In planning the experiment, it was found that for the population ES values which were posited, at $\mathbf{a} = .01$, power would

be very large. This is, however, not relevant to the significance-testing procedure. Assume that the $\mathbf{F}_s$ is found to equal 4.912. What is the status of the null hypotheses on the three population means? The relevant specifications are

$$\mathbf{a} = .01, \quad \mathbf{u} = \mathbf{k} - 1 = 2, \quad \mathbf{n} = 200, \quad \mathbf{F}_s = 4.912.$$

Table 8.3.2 (for $\mathbf{a} = .01$ and $\mathbf{u} = 2$) with row $\mathbf{n} = 200$ yields $\mathbf{F}_c = 4.642$. Since $\mathbf{F}_s$ exceeds this value, the null hypothesis is rejected, and it is concluded (at $\mathbf{a} = .01$) that the three population means are not all equal. Note that one does *not* conclude that the population ES of the power specifications (in this case there were two values, $\eta^2 = .05$ and .10, or $\mathbf{f} = .23$ and .33) necessarily obtains. In fact, the *sample* η^2 is $\mathbf{uF}_s/[\mathbf{uF}_s + (\mathbf{u}+1)(\mathbf{n}-1)] = .016$ and the best estimate of the population η^2 is .013 ($=\epsilon^2$). See section 8.2.2 above and Cohen (1965, pp. 101–106 and ref.).

8.5.2 SIGNIFICANCE TESTING IN CASE 1: **k** MEANS WITH UNEQUAL **n**. When the sample **n**'s are not all equal, the significance testing procedure is as in Case 0 except that one enters the table with their arithmetic mean, i.e., **N**/**k** [formula (8.3.3)]. This will generally not yield a tabled value of **n**, but the **n** scale is such that on the rare occasions when it is necessary, linear interpolation between $\mathbf{F}_c$ values is quite adequate.

Illustrative Examples

8.18 Example 8.3 described an opinion poll on government centralism on a college campus in which there would be a comparison among means of $\mathbf{k} = 6$ academic area groups of unequal size, with a total sample size of approximately 300. The **F** test is to be performed at $\mathbf{a} = .05$. Assume that when the survey is concluded, the actual total $\mathbf{N} = 293$, and $\mathbf{F}_s = 2.405$. Since $\mathbf{N} = 293$, the **n** needed for entry is $\mathbf{N}/\mathbf{k} = 293/6 = 48.8$. What is the status of the null hypothesis of equal population means, for these specifications, i.e.,

$$\mathbf{a} = .05, \quad \mathbf{u} = \mathbf{k} - 1 = 5, \quad \mathbf{n} = 48.8, \quad \mathbf{F}_s = 2.405.$$

In Table 8.3.16 (for $\mathbf{a} = .05$, $\mathbf{u} = 5$) see column $\mathbf{F}_c$. There is no need for interpolation, since, using the conservative **n** of 48, $\mathbf{F}_c = 2.246$, which is exceeded by $\mathbf{F}_s = 2.405$. Therefore, the null hypothesis is rejected, and it can be concluded that the academic area population means on the centralism index are not all equal. (Note again the irrelevance to conclusions about the null hypothesis of the alternate-hypothetical ES of the power analysis described in example 8.3.)

8.19 In example 8.4, samples of varying **n** of psychiatric nurses from $\mathbf{k} = 12$ hospitals were to be studied with regard to differences in mean scores on an attitude scale of Social Restrictiveness towards psychiatric patients. The total $\mathbf{N} = 326$, so the average **n** per hospital is $\mathbf{N}/\mathbf{k} = 27.2$. The significance criterion is $\mathbf{a} = .05$. When the data are analyzed, the $\mathbf{F}_s$ of the test of $\mathbf{H}_0$: $\mathbf{m}_1 = \mathbf{m}_2 = \ldots = \mathbf{m}_{12}$ equals 3.467. The specifications for the significance test, thus, are

$$\mathbf{a} = .05, \qquad \mathbf{u} = \mathbf{k} - 1 = 11, \qquad \mathbf{n} = 27.2, \qquad \mathbf{F}_s = 3.467.$$

There are no tables for $\mathbf{u} = 11$. Although we can linearly interpolate between $\mathbf{F}_c$ values for $\mathbf{u} = 10$ and $\mathbf{u} = 12$ to find $\mathbf{F}_c$ for $\mathbf{u} = 11$, it would only be necessary to do so if $\mathbf{F}_s$ fell between these two $\mathbf{F}_c$ values. The $\mathbf{F}_c$ value for the smaller **u** (here 10) will always be larger than that of the larger **u** (here 12). Thus, if $\mathbf{F}_s$ exceeds the $\mathbf{F}_c$ for $\mathbf{u} = 10$, it must be significant, and if $\mathbf{F}_s$ is smaller than $\mathbf{F}_c$ for $\mathbf{u} = 12$, it must be nonsignificant. Accordingly, we use Table 8.3.19 (for $\mathbf{a} = .05$, $\mathbf{u} = 10$) with row $\mathbf{n} = 27$, and find $\mathbf{F}_c = 1.864$. Since $\mathbf{F}_s = 3.467$ is greater than this value, we conclude that the null hypothesis is rejected at $\mathbf{a} = .05$. Again we call to the reader's attention that we do *not* conclude that the population ES used in the power analysis of example 8.4 necessarily obtains (Cohen, 1973). That value was $\mathbf{f} = .25$, hence (Table 8.2.2) the population η^2 posited was .0588. For the sample, η^2 is .1083 and ϵ^2, the best estimate of the population η^2, is .0771 (Section 8.2.2).

CHAPTER 9

Multiple Regression and Correlation Analysis

9.1 Introduction and Use

During the past decade, under the impetus of the computer revolution and increasing sophistication in statistics and research design among behavioral scientists, multiple regression and correlation analysis (MRC) has come to be understood as an exceedingly flexible data-analytic procedure remarkably suited to the variety and types of problems encountered in behavioral research (Cohen & Cohen, 1983; Pedhazur, 1982; McNeil, Kelly & McNeil, 1975; Ward & Jennings, 1973). Although long a part of the content of statistics textbooks, it had been relegated to the limited role of studying linear relationships among quantitative variables, usually in the applied technology of social science. For example, in psychology it was largely employed in the forecasting of success or outcome using psychological tests and ratings as predictors in personnel selection, college admission, psychodiagnosis, and the like. In its "new look," fixed model MRC is a highly general data-analytic system that can be employed whenever a quantitative "dependent variable" (**Y**) is to be studied in its relationship to one or more research factors of interest, where each research factor (**A**, **B**, etc.) is a *set* made up of one or more "independent variables" (IVs). The form of the relationship is not constrained: it may be straight-line or curvilinear, general or